Efficient Risk Sharing with Risky Durables

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September, 2006

Abstract

This paper analyzes efficient risk sharing contracts between a cost-minimizing financial intermediary and heterogeneous households who face idiosyncratic income risk and potential catastrophic risk to their durable goods. The intermediary is committed to the contract while households may walk away from the contract in any period with punishment. The key theoretical contribution of the model is the introduction of endogenous state-dependent autarky into an one-sided commitment framework with a continuum of agents. An individual's value of autarky depends on two components, an exogenously determined level of endowment and an endogenously determined level of durable holdings. The main finding is that even in the presence of catastrophic durable risk, the efficient contract exhibits substantial though not perfect risk sharing. When compared with CEX data, this paper finds that this environment accounts for only a small portion of the observed incomplete risk sharing. However, regional differences in the level of risk and amount of risk sharing appear consistent with the results of one-sided commitment.

JEL Classifications: D52, D61, G22.

*I would like to thank Don Schlagenhauf, Eric Young and Carlos Garriga for their help on this project. Any remaining errors are my own.

1 Introduction

Hurricanes, earthquakes, floods, or terrorist attacks can result in catastrophic losses to durables. These low probability events can lead to large financial burdens for households and the insurance industry, exemplified by the events surrounding the aftermath of Hurricane Andrew. Hartwig (2002) documents that the financial consequences of this storm spread far beyond the $34 billion in structural damage and $20 billion in insured losses. Eleven insurance companies filed for bankruptcy following Andrew. Many other insurance companies lost a substantial portion of their surpluses. Insurance companies were unable to fulfill 700,000 claims filed because of the storm. These shortfalls resulted in the state of Florida and Dade County absorbing some of these catastrophic losses. Immediately following the hurricane, the state of Florida issued $500 million in bonds to cover the immediate costs of the storm. Andrew also brought about additional costs which are still accumulating today.[1]

To get a sense of the risks associated with durable goods, Table 1 displays the ten most expensive, in terms of insured losses, catastrophic events to have ever occurred in the US. The table only reports insured losses from the largest events. These amounts do not include the costs of structural fires which typically cost between $11 and $13 billion a year, the cost of tornados which is roughly $5 billion a year, and the cost incurred by the roughly 4 million automobile crashes that occur annually. The aforementioned events and growing reinsurance markets suggest the current mechanisms used to share risk between insurance companies and households might not completely eliminate the risk associated with the loss of durable goods and catastrophic events.

This paper analyzes efficient risk sharing contracts with a cost-minimizing financial intermediary and heterogeneous households who face idiosyncratic income risk and potential catastrophic risk to their durable goods. Just as

[1] These costs include the funding of an $11 billion Hurricane Trust Fund, mandated 2% windstorm deductibles for homeowners, increased local taxes, and stricter building codes. A detailed list of the lasting financial effects of Hurricane Andrew can be found in Hartwig (2002).

with many insurance contracts, the intermediary is committed to the insurance contract while households may walk away from the contract in any period. Thus, this is a problem of limited commitment. The financial intermediary must offer contracts that make the households prefer to remain in the risk-sharing contract rather than walking away and living in autarky. Commitment problems have been previously studied in Kehoe and Levine (1993, 2001) and others. Kehoe and Perri (2001), and Seppala (1999) study one-sided commitment problems with endogenous autarky and two types of agents. Krueger (1999) and Krueger and Perri (1999) study one-sided commitment problems with a continuum of agents and exogenous autarky. The key theoretical contribution of our model is the introduction of endogenous autarky into the one-sided commitment framework with a continuum of agents. Krueger and Ulhig (2005) and Phelan (1995) study commitment problems where the outside option is endogenously determined by the value of contracts offered by competing intermediaries. In this paper, an individual's value of autarky depends on two things, the exogenously determined level of endowment and the endogenously determined level of durable holdings. Thus, autarky is dependent on an individual's wealth.

In a numerically solved dynamic contracting model, the findings are that even in the presence of this catastrophic durable risk, the efficient contract exhibits substantial though not perfect risk sharing. This means that even under the efficient arrangement, some households will suffer losses in the event of a catastrophic shock. Given a belief that this is world of one-sided commitment, this has important implications regarding government insurance programs which provide complete insurance in the aftermath of a catastrophic event. This model would imply that these programs are not implementable by the private sector.

Another objective is to compare the risk sharing found in the limited commitment model with that found in empirical data. Few papers have addressed the empirical implications of limited commitment. Only a few papers provide direct empirical evidence of limited commitment. Among these papers are Attanasio and Ríos-Rull (2000), Foster and Rosenzweig (1999), Ligon, Thomas and Worrall (2002), and Krueger (1999). The first three papers examine the level of risk sharing in undeveloped villages. They find that limited commitment does a good job at explaining the amount of risk sharing which is inherit in contracts written throughout these villages. Enforcement problems inhibit complete insurance even in these simple economies. In the paper which this paper most resembles, Krueger (1999) showed that a limited commitment economy displays a high degree of risk sharing that closely matches non-durable expenditure patterns in CEX data. The key difference now is the disaggregation of expenditures into both non-durables and durables. It is not immediately clear how the introduction of risky durables will affect the risk-sharing in this economy. Owners of durable goods face a tension. On one hand, as the household increases its durable holdings, it faces a larger downside risk in utility if a catastrophic loss were to occur. Across this margin, we would expect that households with larger durables would find it welfare improving to remain in a risk-sharing environment and as a result the economy would exhibit more risk sharing. On the other hand, households with larger durables also have the ability to self-insure. They could use their current durable stock as a buffer to income risk. Given that a household does not experience a catastrophic durable shock, if a bad income shock occurs, they can simply sell off part of their durable stock to smooth consumption. This paper can address whether risky durable goods enhance or hinder risk sharing.

This paper finds that the introduction of endogenous autarky does not distort the two typical results of a one-sided commitment model. First, the one-sided commitment model displays substantial though not perfect risk sharing. Second, the one-sided commitment model displays substantially more risk sharing than an exogenous borrowing constraint model. However, when compared to CEX cross-sectional data, we find that both models display too much risk-sharing in consumption and durable expenditures. Thus, one-sided commitment accounts for only part of the incomplete risk sharing we see in the data. The self-insure aspect of durables is substantial and some additional mechanism must also be hindering full risk sharing.

The paper is organized into six sections. The first section describes the economic environment. The second section defines and characterizes efficient allocations in this environment. The third section presents the model of one-sided commitment and characterizes the efficient risk sharing arrangement implied by this model. The efficient allocations are compared with the possibilities of perfect risk sharing and autarky. A decentralization methodology is also proposed. The fourth section shows how the solution to the baseline model compares to the exogenous borrowing model. The fifth section compares the model with CEX data. The final section presents conclusions and possible extensions. The paper also contains an appendix with proofs of theoretical results and a description of the computational method used to solve the one-sided commitment model.

2 The Environment

This section describes the environment used to characterize efficient allocations. The economy is populated by a continuum of households that have preferences over non-durable and durable consumption streams given by

$$U(\{c_t, d_{t+1}\}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, d_{t+1}).$$

The period utility function u is assumed to be strictly increasing, strictly concave, twice differentiable, and satisfies the Inada conditions. The functional form of u is chosen to be

$$u(c_t, d_{t+1}) = \log(c_t) + \theta \log(d_{t+1})$$

which satisfies all of these assumptions and is consistent with the fact that the shares of durable expenditures and non-durables expenditures in household spending has been relatively constant even with the rapid fall in durable prices.

Households consume two goods in this economy: non-storable consumption, c, and durables, d. Households face a stochastic endowment process $\{y_t\}$ which follows a Markov process with transition probabilities $\pi(y'|y)$. The transition probabilities are invariant across the population. From the law of large numbers, the transition probability $\pi(y'|y)$ equals the fraction of the households who experience the shock y' given they have endowment y today. In addition, households face the risk of a catastrophic loss to their durables. This shock occurs with some small probability π_f[2]. We can combine these two shocks into a new process $\{s_t\}$ which follows a Markov process with probabilities $\pi\left(s'|s\right)$. From the law of large numbers, we find this transition probability $\pi\left(s'|s\right)$ equals the fraction of households who experience a particular combination of endowment and durable shocks, s' given they enter in state s. The transition $\pi\left(s'|s\right)$ is assumed to have a unique invariant distribution Ψ.

Denote $s_t \in S$ as the current household state and $s^t = (s_0, s_1, ..., s_t)$ as the history of shock realizations. The probability of getting to any history, s^t at time t is

$$\pi\left(s^t|s_0\right) = \pi(s_t|s_{t-1})\pi(s_{t-1}|s_{t-2})...\pi(s_1|s_0).$$

It is assumed that at date 0 that the distribution over current states is given by Ψ. This implies that there is no aggregate uncertainty in this model.

Consumers trade in a market consisting of a complete set of state contingent assets. They receive utility from two types of goods: non-durable goods and durable goods. In the initial period, every household is distinguished by their initial asset holdings, a_0, durable holdings, d_0, and their initial shock state, s_0. A household's initial asset holdings gives them claims to consumption and durable investment. An allocation$\left\{c_t\left(a_0, d_0, s^t\right),\ d_{t+1}\left(a_0, d_0, s^t\right)\right\}$ then specifies how much an agent of type $\left(a_0, d_0, s^t\right)$ consumes and how many durables they buy, conditional on arriving to the period with history s^t. Markets in this model are endogenously incomplete in that individuals optimally choose whether or not to participate in the market. The households have the option of opting out of the claims market and defaulting on

[2]This implies that durable shocks are idiosyncratic at the household level. This interpretation of durable shocks seems appropriate for several events like tornados, house fires car accidents, and terrorist attacks. This may or may not be entirely true for large scale events like hurricanes or earthquakes. From a national perspective, the actual area of devastation from a hurricane or earthquake is actual very localized near the center of the event. So, in some ways these event take on idiosyncratic characteristics.

contracts. The punishment from such behavior is that agents who opt out may never rejoin the market in future and live in autarky. Just as with current US bankruptcy law, current durable holdings can not be seized. Under autarky, households are forced to consume and buy future durables from their endowment and existing durable stock in each period. The use of this type of mechanism implies there must exist some central agency which keeps track of who has defaulted in the past. We know that households will not default on an allocation if and only if for each possible state of the world in the future, the expected discounted utility from continuing in the market is at least as high as living in autarky. Denoting the current period utility of an allocation $\left\{c_t\left(a_0, d_t, s^t\right), d_{t+1}\left(a_0, d_t, s^t\right)\right\}$ as u_t, an individual will not default if

$$u_t + \sum_{\tau > t} \sum_{s^\tau | s^t} \beta^{\tau - t} \pi(s^\tau | s^t) u_\tau \geq u_t^{AUT} + \sum_{\tau > t} \sum_{s^\tau | s^t} \beta^{\tau - t} \pi(s^\tau | s^t) u_\tau^{AUT} \qquad \forall s^t$$

where u_t^{AUT} represents the period t utility a household receives from an allocation

$$\left\{c_t^{AUT}\left(a_0, d_t, s^t\right),\ d_{t+1}^{AUT}\left(a_0, d_t, s^t\right)\right\}$$

which is funded solely from the household's current endowment, durable stock, and asset holdings.

In this model, default will not occur in equilibrium. In the absence of private information, no individual would be willing to write a contract with a household where in any given state default would occur.

3 Efficient Allocations

This section defines efficient allocations for this environment. This discussion will use ideas and results developed in Atkeson and Lucas (1992, 1995) and extended in Krueger and Perri (1999). The main methodological contribution of Atkeson and Lucas (1992) is to analyze the problem of finding efficient allocations in terms of state contingent utilities instead of state contingent consumption. The previous section described several functions which were history dependent. The transformation from the history dependent primal problem into the dual problem makes the problem computationally feasible. Households can be characterized by their current state instead of their entire history.

Since we are in an environment where agents can walk away from a risk sharing arrangement, the intermediary faces constraints in that the agents who wish to leave the arrangement must be enticed, through higher utility, not to leave the risk sharing arrangement. There exists some lower bound on utility where in some given state, households are indifferent between staying in the risk sharing arrangement and living off their endowment. The planner faces state contingent participation constraints which define the lower bound on utilities that keep households in the risk sharing arrangement. Instead of being indexed by initial assets and shocks, households are now indexed by their entitlement to lifetime utility at period 0, w_0, the initial durable holdings d_0, and their initial state s_0, where the state of the household is determined by their combination of income and durable shocks.

Let μ_0 be the initial period joint measure over (w_0, d_0, s_0), and let s^t denote the sequence of shocks from periods 0 to t. An allocation is defined as a sequence $\{h_t(w_0, d_t, s^t)\}_{t=0}^{\infty}$ that maps initial utility entitlements w_0 and shock sequences, s^t, into levels of current period t utility. Here the relationship between h_t and the current period utility derived from c_t and d_{t+1} is given by $h_t(w_0, d_t, s^t) = u(c_t(w_0, d_t, s^t), d_{t+1}(w_0, d_t, s^t))$. Thus, $h_t(w_0, d_t, s^t)$ is the current period utility that an agent of type (w_0, d_0, s_0) receives if they experience the sequence of shocks s^t and have durable holdings of d_t entering period t. With this definition we can now define constrained feasibility and efficiency in this environment.

For any allocation $h = \{h_t(w_0, d_t, s^t)\}_{t=0}^{\infty}$ define

$$U_t(w_0, d_t, s^t, h_t) = h_t(w_0, d_t, s^t) + \sum_{v>t}^{\infty} \sum_{s^v} \beta^{v-t} \pi(s^v) h_v(w_0, d_v, s^v) \qquad (1)$$

and

$$U_t^{AUT}(y_t, d_t) = u(y_t, d_t) + \sum_{v>t}^{\infty} \sum_{s^v} \beta^{v-t} \pi(s^v) u(y_v, d_v). \quad (2)$$

The first equation defines the continuation utility from an allocation h for an agent of type $(w_{0,}d_0, s_0)$ from period t and shock history s^t forward. The second equation defines a similar continuation utility from autarky for the same agent type.[3] Feasible allocations can now be defined as follows.

Definition 1 *An allocation $h = \{h_t(w_0, d_t, s^t)\}_{t=0}^{\infty}$ is* ***constrained feasible*** *with respect to a joint distribution over utility entitlements and initial endowments, μ_0, if for each (w_0, d_0, s_0)*

$$w_0 = U_0(w_0, d_0, s_0, h), \quad (3)$$

$$U_t(w_0, d_t, s^t, h) \geq U_t^{AUT}(y_t, d_t) \forall s^t, \quad (4)$$

$$\lim_{t \to \infty} \beta^t \sup_{s^t} U_t(w_0, d_t, s^t, h) = 0, \quad (5)$$

and

$$\sum_{s^t} \int (c_t + d_{t+1} - d_t - y_t) \pi(s^t | s_0) d\mu_0 \leq 0 \;\forall t. \quad (6)$$

Equation (3) is a promise keeping constraint. The allocation gives utility w_0 to agents that are entitled to w_0. Equation (4) represents the set of participation constraints for this environment. The continuation utility generated by the allocation is at least as high as the continuation utility generated from autarky. Equation (5) is a boundedness condition on utility that ensures that the discounted continuation value goes to zero in the limit. Equation (6) is the resource feasibility constraint for this environment. It requires that the summation of consumption goods plus new durable expenditures in every period must be less than or equal to the aggregate endowment in the economy.

[3] Spear and Srivastava (1987) prove that the continuation utility of an agent is sufficient to summarize the agent's history.

We now define the idea of efficiency in this environment in a way similar to Atkeson and Lucas (1995).

Definition 2 *An allocation* $h = \{h_t(w_0, d_t, s^t)\}_{t=0}^{\infty}$ *with sequences* $\{c_t, d_{t+1}\}_{t=0}^{\infty}$ *is **efficient** with respect to* μ_0 *if*

1. *It is constrained feasible with respect to* μ_0.
2. *There does not exist another allocation* $\hat{h} = \{\hat{h}_t(w_0, d_t, s^t)\}_{t=0}^{\infty}$ *with sequences* $\{\widehat{c}_t, \widehat{d}_{t+1}\}_{t=0}^{\infty}$ *that is constrained feasible with respect to* μ_0 *and such that for some* t

$$\sum_{s^t} \int (\{\widehat{c}_t + \widehat{d}_{t+1} - d_t)\pi(s^t|s_0)d\mu_0 < \sum_{s^t} \int (c_t + d_{t+1} - d_t)\pi(s^t|s_0)d\mu_0$$

The definition states that an allocation is efficient if it attains utility promises made under μ_0 in a rational and resource feasible way, and no other allocation can do so with fewer resources in any period.

4 Model of One-Sided Commitment with Catastrophic Risk to Durable Goods

In this section we describe the method we use to solve for efficient allocations. First,the model is presented. Second, we solve for the two extreme solutions in this environment: autarky and perfect risk sharing. Finally, we present the solution to the baseline model and show that it does not generate the same risk sharing behavior as either autarky or perfect risk sharing.

This economy is formulated as a component planner problem similar in fashion to Atkeson and Lucas (1995) and Krueger and Perri (1999). In this framework, the planner seeks to minimize the costs of distributing goods and promised future utilities across the economy given a fixed aggregate endowment. From this point forward we use the prime notation to denote next period variables. In our economy with two income states and two durable states, the planner chooses consumptions, c_s, durables, d'_s, and state contingent future utilities, $w_s^1, w_s^2, w_s^3, w_s^4$, that solve the following minimization problem

$$V(w,d,s) = \min_{c_s, d'_s, w_s^1, w_s^2, w_s^3, w_s^4} \begin{matrix} [c_s + d_s^{'} - d + q[(1-\pi_f)\pi_{s,1}V(w_s^1, d'_s, 1) + \\ (1-\pi_f)\pi_{s,2}V(w_s^2, d'_s, 2) \ + \pi_f \pi_{s,1} V(w_s^3, 0, 1) + \\ \pi_f \pi_{s,2} V(w_s^4, 0, 2)]] \end{matrix} \quad (7)$$

where π_f denotes the probability of a catastrophic loss of durables, and $\pi_{s,s'}$ represents the probability of moving from income state s to income state s'. Given the planner faces a limited endowment, we can think of the price q as representing the planner's price on future utility relative to current utility. In the preceding description of future utilities w_s^i, the value of i can be $1, 2, 3$, or 4 depending on the agent's realization of income and durable shocks. Using this notation, $i = 1$ represents the state of the world where the agent receives high income, and maintains their durable stock. Agents also keep their durables when $i = 2$; they simply receive low income instead of high income. When $i = 3$, the agent realizes high income, but also experiences a catastrophic shock to their durables. In this case the agent's entire stock of durables is wiped out. This same scenario is true when $i = 4$, only the agent receives a low income shock.

The planner also faces a one-sided commitment problem which introduces two additional constraints into the optimization problem. First, since the planner is committed to the risk sharing arrangement, the planner must honor any promised utilities in the future. In other words, the planner faces a promise keeping constraint of the form

$$w = U(c_s, d'_s) + \beta \left[(1-\pi_f)\pi_{s,1} w_s^1 + (1-\pi_f)\pi_{s,2} w_s^2 + \pi_f \pi_{s,1} w_s^3 + \pi_f \pi_{s,2} w_s^4 \right] \quad (8)$$

for all s. Second, the planner faces the possibility of agents leaving the risk-sharing environment. To ensure that all agents choose to remain in the risk sharing arrangement, the planner needs to ensure that the utility generated under the risk sharing environment is always at least as good as the autarky. Thus, agents should always weakly prefer to remain in the risk sharing environment versus self-insuring under autarky. Given this condition, the planner faces the following participation constraints:

$$w_s^1 \geq V^{AUT}(d'_s, 1), w_s^2 \geq V^{AUT}(d'_s, 2) \tag{9}$$

and

$$w_s^3 \geq V^{AUT}(0, 1), w_s^4 \geq V^{AUT}(0, 2) \tag{10}$$

for all s, where $V^{AUT}(d'_s, s)$ is the utility of a household who leaves the risk sharing arrangement and lives solely off their endowment and existing stock of durables each period.

4.1 Solution to Autarky

In autarky, the household chooses consumption, c, and durables, d', subject to the constraint that all consumption and durable purchases are financed through the household endowment and existing stock of durables. The autarky consumption/savings problem is formulated as

$$\begin{aligned} V^{AUT}(d, s) &= \max_{c,d'} U(c_s, d'_s) + \pi_{s,1}(1 - \pi_f)\beta V^{AUT}(d'_s, 1) + \\ &\quad \pi_{s,2}(1 - \pi_f)\beta V^{AUT}(d'_s, 2) + \pi_{s,1}\pi_f\beta V^{AUT}(0, 1) + \pi_{s,2}\pi_f\beta V^{AUT}(0, 2) \end{aligned}$$

subject to

$$y_s \geq c_s + d' - d.$$

Since the household budget constraint is always binding, the problem simplifies to a choice of only d' and (4.7) can be written as

$$\begin{aligned} V^{AUT}(d, s) &= \max_{d'} U(y_s + d - d'_s, d'_s) + \pi_{s,1}(1 - \pi_f)\beta V^{AUT}(d'_s, 1) + \\ &\quad \pi_{s,2}(1 - \pi_f)\beta V^{AUT}(d'_s, 2) + \pi_{s,1}\pi_f\beta V^{AUT}(0, 1) + \pi_{s,2}\pi_f\beta V^{AUT}(0, 2) \end{aligned}$$

This is a straightforward dynamic programming problem. The solution to the autarky problem is displayed in Figure 1. The figure shows an obvious result, the value of autarky increases as durable holdings increase. Thus, the planner is going to be forced to give agents with higher durable stocks greater promised utility as these agents have a greater incentive to leave the risk sharing arrangement.

4.2 Solution to Perfect Risk Sharing

In this section, we are going to characterize the perfect risk sharing allocation. Under perfect risk sharing, the agent's utility is constant across all realizations of income and durable shocks. Thus we can think of the perfect risk sharing arrangement as a solution to our existing economy except that the promise utilities must be the same across all states over time. Therefore, $w_s^1 = w_s^2 = w_s^3 = w_s^4 = w$. Thus, the planner's problem can be written as

$$V(w,d,s) = \min_{c_s, d'_s} \begin{array}{c} [c_s + d'_s - d + q[(1-\pi_f)\pi_{s,1}V(w,d'_s,1) + (1-\pi_f)\pi_{s,2}V(w,d'_s,2) \\ +\pi_f\pi_{s,1}V(w,0,1) + \pi_f\pi_{s,2}V(w,0,2)]] \end{array}$$

subject to promise keeping constrain:

$$w = U(c_s, d'_s) + \beta\left[(1-\pi_f)\pi_{s,1}w + (1-\pi_f)\pi_{s,2}w + \pi_f\pi_{s,1}w + \pi_f\pi_{s,2}w\right]$$

and participation constraints

$$w \geq V^{AUT}(d'_s, 1), w \geq V^{AUT}(d'_s, 2)$$

$$w \geq V^{AUT}(0, 1), w \geq V^{AUT}(0, 2).$$

Since the value of autarky is increasing in both the durable state and endowments, the last three participation constraints can never be binding under perfect risk sharing. Thus, the first order conditions for perfect risk sharing allocation are

$$1 - q - \frac{c\theta}{d'_s} = 0 \tag{11}$$

and

$$w(1-\beta) = \log(c) + \theta\log(d'_s). \tag{12}$$

These equations allow us to solve for c and d'_s analytically. By combining the two equations and using the fact that $q = \beta$ under perfect risk sharing we find that

$$d'_s = \exp\left(-\frac{\ln(1-\beta) - \ln\theta - w + w\beta}{1+\theta}\right)$$

and

$$c = \exp\left(-\frac{-w + w\beta - \theta\ln(1-\beta) + \theta\ln\theta}{1+\theta}\right).$$

This shows that the perfect risk sharing allocation is invariant to household's current stock of durables, d. In addition, the policy functions d'_s and c are strictly increasing in w. Figure 2 presents the optimal policy functions under perfect risk sharing for a given parameterization with $\beta = 0.97$ and $\theta = 0.068$. The only stationary distribution in this environment is degenerate to a single w. Thus, at steady-state all agents have identical utility w given by (12).

4.3 Solution to One Sided Commitment

Given that we have identified the range of the possible risk sharing arrangements in this environment we are now going to present the qualitative features of the efficient allocation. Given the same parameterization as in the previous section, Figures 3 and 4 display the typical shape of promise utilities. Figure 3 displays promise utilities as a function of existing durable holdings. If possible, the planner seeks to maintain the current level of utility if allowed by the participation constraint. Since, the value of autarky is an increasing function in durables, as a household's stock of durables rises, participation forces the planner to offer higher levels of promise utility. The wealthier households must be enticed not to default from the risk sharing arrangement. Figure 4 displays promise utilities as a function of current utilities. The figure displays promise utilities for a household with durable stock $\widehat{d}$ in an economy with two durable choices, d and d^* such that $d^* > d$. The promise utilities for durable level d are represented by dashed lines. The promise utilities for durable level d^* are represented by solid lines. Once again, we see that unless necessary, promise utilities are held constant. For example, $w^2(d, 2)$ is a flat line through the 45° line. At some point, the promise utility begins to rise parallel to the right of the 45°. The region A represents the level of current utility. Note that promise keeping implies that

$$w = U(c, d') + \beta w'$$

thus the region A is equal to $U(c, d')$. Once $w^2(d, 2)$ reaches the right edge of A, any increase in w must be matched with an equal increase in $w^2(d, 2)$. This same argument holds when we look at $w^2(d^*, 2)$. Since $d^* > d$, we know $V^{AUT}(d^*, 2) > V^{AUT}(d, 2)$ and $w^2(d^*, 2) > w^2(d, 2)$. Notice that $w^2(d^*, 2)$ begins to parallel the 45° line further to the right. The higher durable stock entitles this household to a larger current utility, and therefore, region B is wider than region A. The thicker line in Figure 4 represents the decision rules for promise utilities. Policy functions are conditioned only on the state of the world $\widehat{d}$, not the choice of future durables d or d^*. We observe several characteristics of these functions. First, each value of promise utility, $w'_{s'}$, corresponding to state s', is greater than or equal to the corresponding autarky value, $V^{AUT}(s')$. This result is follows directly from the participation constraints. Second, the promise utilities are constant whenever $w \geq V^{AUT}(s')$. It appears that whenever the participation constraint is not binding, utility is held constant. Third, the lowest promise value denoted W4H crosses the 45 degree line at exactly the lowest value of autarky. Also, for any current utility greater than or equal to the lowest autarky value, $w \geq V^{AUT}(0, 2)$, the rules are either constant at autarky or increasing in current utility, w. Across the income states the promised utility for high income states is greater than or equal to promised utility for low income state, $w_s^{s'(y=y_H)} \geq w_s^{s'(y=y_L)}$, at any current utility w and durable stock, d. Across the durable states, the promised utility in states that suffer no catastrophe is greater than or equal to promised utility for states when a catastrophic shock occurs, $w_s^{s'(d'>0)} \geq w_s^{s'(d'=0)}$, at any current utility w and durable stock d. Figure 5 displays the typical decision rules for consumption and durables for the same parameterization used earlier. As expected both nondurable and durable consumption increase with current utility. Now that we have seen an example of the policy functions, we need to characterize these functions more generally.

4.3.1 Existence of optimal rules for given price q

We show the existence of optimal allocation rules by solving the planner's problem with additional constraints. We characterize the allocations from this problem and show that the additional constraints are not binding. Thus, the solution to this new problem is identical to the original planner's problem.

Consider the planner's problem given by equations (7) through (10). We simply modify the original problem with the additional constraint $w \leq \hat{w}$, which bounds the promised utility. The bounded Bellman equation will be defined on $C(\Omega)$, the space of continuous and bounded functions on Ω where $\Omega = \left\{ w | \min_y U^{AUT}(y,0) \leq w \leq \hat{w} \right\}$ is contained in a compact subset of a real space D. Consider the operator T_q defined as

$$T_q V(w,d,s) = \min_{c_s, d'_s, w^1_s, w^2_s, w^3_s, w^4_s} \left[\begin{array}{c} c_s + d'_s - d + \\ q \left[\begin{array}{c} (1-\pi_f)\pi_{s,1} V(w^1_s, d'_s, 1) + (1-\pi_f)\pi_{s,2} V(w^2_s, d'_s, 2) \\ +\pi_f \pi_{s,1} V(w^3_s, 0, 1) + \pi_f \pi_{s,2} V(w^4_s, 0, 2) \end{array} \right] \end{array} \right]$$

subject to

$$w = U(c_s, d'_s) + \beta \sum_{s' \in S} \pi(s'|s) w^{s'}_s$$

and

$$V^{AUT}(d', s') \leq w^{s'}_s \leq \hat{w} \text{ for all } s'$$

where $d' = 0$ if a catastrophic event occurs. The final inequality in the participation constraint is the only difference between this problem and the original planning problem. Now consider two extreme cases.

Case 1: $w^{s'}_s = \hat{w}$ for all $s.'$.

In this case the two constraints come together to generate a lower bound on $U(c_s, d'_s)$. Denote $h_L(w,d,s)$ to be this lower bound. Combining the constraints we find

$$h_L(w,d,s,) = \frac{w - \beta\hat{w}}{(1-\beta)}$$

Case 2: The participation constraint always binds, $w_s^{s'} = V^{AUT}(y', d')$.

In this case the two constraints come together to form an upper bound on $U(c_s, d'_s)$. Denote $h_U(w,d,s)$ to be this upper bound. Combining the constraints we find

$$h_U(w,d,s) = \frac{w - \beta\sum_{s'\in S}\pi(s'|s)V^{AUT}(y', d')}{(1-\beta)}$$

where $h_U(w)$ bounded only if $V^{AUT}(y', d')$ is bounded for all y' and d'. It is sufficient to show that there exists a d_U such that for any $d \geq d_U$, $d' \leq d$. This is the key theoretical result that extends the results of Krueger (1999) to the endogenous autarky present in the environment.

Proposition 3 *There exists a d_U such that, under autarky, for any $d \geq d_U$, $d' < d$.*

Proof. See Appendix. ■

This upper bound on d' makes the set of feasible allocations compact. Given the additional constraint and the upper bound on d' we can construct our bounded dynamic programming problem on the interval $h_L(w,d,s) \leq h \leq h_U(w,d,s)$. All that needs to be shown is that T_q can be characterized as a contraction mapping.

Lemma 4 *For $q < 1$, T_q maps $C(\Omega)$ into itself and is a contraction mapping.*

Proof. See Appendix ■

4.3.2 Characterization of Value Function and Policy Functions

We now proceed to characterize the value function V, the fixed point of T_q and the optimal policy functions. We begin by showing that value function is strictly increasing with respect to w, and decreasing with respect to d.

Lemma 5 *The value function, V, is increasing in w, decreasing in d, and convex in both arguments.*

Proof. See Appendix. ■

We are going to rely heavily on the first order conditions of this problem to characterize the policy functions. The first order necessary conditions are

$$\begin{aligned}
0 &= 1 + q \sum_{s' \in S} \pi(s'|s) V'_{d'}(w', d', s') + \mu_{s'}(V_{d'}^{AUT}(d', s')) \\
0 &= q\pi(s'|s) V'_{w^{s'}}(w^{s'}, d', s') - \frac{\beta \pi(s'|s)}{U_c(c, d')} - \mu_{s'} - \phi_{s'} \text{ for all } s' \in S \\
w &= U(c_s, d'_s) + \beta \sum_{s' \in S} \pi(s'|s) w_s^{s'} \\
V^{AUT}(d', s') &\leq w_s^{s'} \leq \hat{w}, \text{ for all } s' \in S
\end{aligned}$$

where $\mu_{s'}$ represents the corresponding multiplier on the participation constraint, and $\phi_{s'}$ represents the corresponding multiplier on the upper bound constraint on promise utilities.

The envelope conditions for d and w are

$$\begin{aligned}
V_d(w, d, s) &= -1, \\
V_w(w, d, s) &= \frac{1}{U_c(c, d')} \\
V_d^{AUT} &= U_c(c, d').
\end{aligned}$$

The envelope condition from the autarky problem is embedded in the first order condition on d'; this is where the endogenous autarky may have implications for the solution to the problem.

We are ready to characterize the optimal policy functions. First we describe the relationship between c, d', and w' and w. At a given level of durables, as current utility increases, as Figures 3 and 4 show all choices c, d', and w' also increase. The only constraints that keeps agents from perfectly smoothing away risk are the participation constraints. If binding, certain agents have to be promised more than the optimal amount of promised utility to prevent them from reverting to the autarky allocation. The fact that the choices c and d' increase with w is not surprising given the assumptions placed on the form of the utility function and an inherit nature for agents wishing to keep a mixture of consumption and durables in their portfolio. As an agent's current utility grows they need to be compensated with more consumption and durables. This result is trivial from the first order conditions. We now characterize the optimal policy function on promised utility,

Lemma 6 *Given* $V \in C(\Omega)$.

If $w' > V^{AUT}(d', s')$ *and* $\overline{w}' > V^{AUT}(\overline{d}', \overline{s}')$, *then* $w' = \overline{w}'$.

If $w' > V^{AUT}(d', s')$ *and* $\overline{w}' = V^{AUT}(\overline{d}', \overline{s}')$, *then* $w' \leq \overline{w}'$ *and it must be the case that* ($y' \leq \overline{y}'$ *and/or* $d \leq \overline{d}$).

Proof. Comes directly from the first order conditions. ■

Lemma 7 *The optimal policy function* w' *associated with* $V \in C(\Omega)$ *is increasing in* w.

Proof. See Appendix. ■

The promised utilities are equalized whenever the participation constraints permit it. The promise utilities are increased in states where the participation constraint is binding.

Lemma 8 *Given our* $V \in C(\Omega)$. *For every* (w, d, s), *if* $w' > V^{AUT}(d', s')$, *then* $w' < w$. *Furthermore, for each* s', *there exists a* $w_{s'}$ *such that* $w' = w_{s'} = V^{AUT}(d', s')$

Proof. See Appendix. ■

Proposition 9 *There exists a* $\overline{w} \in \Omega$ *such that* $w' < w$ *for every* $w \geq \overline{w}$ *and every* d *and* y'.

Proof. See Appendix. ■

This proposition implies that whenever $w \in \left[\min_y U^{AUT}(y, 0) \leq w \leq \hat{w}\right]$, then for all d and y', the constraint $w' \leq \overline{w}$ is never binding. Thus when the problem is defined over the set Ω the solution to the bounded Bellman equation with the constraint $w' \leq \overline{w}$ has the same solution as the problem without this constraint.

4.3.3 Decentralization

In this section, we describe how to decentralize a stationary efficient allocation $h = \{h_t(w_0, d_t, s^t)\}_{t=0}^{\infty}$ induced by the optimal policy functions from the component planner's problem into a competitive equilibrium. An agent is indexed by his period 0 state, s_0, durables, d_0 and wealth, a_0. We measure period 0 wealth as an entitlement to period 0 consumption and period 1 durables Let Γ_0 be the joint distribution over (a_0, d_0, s_0). In this section, we will consider the decentralization techniques applied in Kehoe and Levine (1993). In their paper, they consider an equilibrium concept consisting of a complete set of contingent claims to future goods which are traded by individuals in period 0. In this paper, individuals will have a access to a complete set of contingent claims to future consumption and durables which are traded in period 0. Thus, except for the nature of the limited commitment constraints, this is a standard Arrow-Debreu equilibrium.

Let $p_t(s^t)$ denote the price at period 0 of a contract which delivers an allocation consisting of one unit of consumption or one unit of next period durables at period t to a person who has experienced shock history s^t. Since individuals can use their endowment for the purchase of consumption or durables, they must have the same price. Walras' Law allows us to normalize the period 0 price, $p_0(s_0) = 1$.

A household type (a_0, d_0, s_0) chooses an allocation $\{c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t)\}$ to solve

$$\max u(c_0(a_0, d_0, s_0), d_0'(a_0, d_0, s_0)) + \sum_{t=1}^{\infty} \sum_{s^t|s_0} \beta^t \pi(s^t|s_0) u(c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t)) \tag{13}$$

subject to

$$c_0(a_0, d_0, s_0) + d_0^{'}(a_0, d_0, s_0) + \sum_{t=1}^{\infty} \sum_{s^t|s_0} p_t(s^t) \left[c_t(a_0, d_0, s^t) + d_t'(a_0, d_0, s^t) - d_t(a_0, d_0, s^t) \right]$$

$$\leq \quad a_0 + d_0 + y_0 + \sum_{t=1}^{\infty} \sum_{s^t|s_0} p_t(s^t) y_t \tag{14}$$

and

$$U_t(a_0, d_0, s^t, c_t, d_t') \geq U_t^{AUT}(y_t, d_t) \tag{15}$$

where the final constraint is simply a participation constraint which states that the continuation utility for any allocation from period t forward must be greater than or equal to the continuation utility generated from autarky. We can now define the equilibrium for this system

Definition 10 *A* ***competitive equilibrium without default*** *consists of prices* $\{p_t(s^t)\}_{t=0}^{\infty}$ *and allocations*

$$\{c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t)\}_{t=0}^{\infty}$$

such that:

i Given prices, the allocations solve the household's problem for all (a_0, d_0, s_0).

ii Markets Clear: For any period t

$$\int \sum_{s^t} c_t(a_0, d_0, s^t) \pi(s^t|s_0) d\Gamma_0 +$$
$$\int \sum_{s^t} \left[d_t'(a_0, d_0, s^t) - d_t(a_0, d_0, s^t) \right] \pi(s^t|s_0) d\Gamma_0$$
$$= \int \sum_{s^t} y_t \pi(s^t|s_0) d\Gamma_0.$$

Equilibrium Prices Let $\beta^t \pi(s^t|s_0) \mu(a_0, d_0, s^t) \geq 0$ be the Lagrange multiplier associated with the participation constraint at history s^t and $\lambda(a_0, d_0, s_0) \geq$

0 be the Lagrange multiplier associated with the period 0 budget constraint. Let $P(s^t) = \{s^\tau | \pi(s^t|s^\tau) > 0\}$ be the set of all shock histories that can have s^t as its continuation value. The first order condition on consumption for the individual's optimization problem in period t and history s^t is

$$\beta^t \pi(s^t|s_0) u_c'(c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t)) \left[1 + \sum_{s^\tau \in P(s^t)} \mu(a_0, d_0, s^\tau) \right] = \lambda(a_0, d_0, s_0) p_t(s^t)$$

Combine these conditions across two time periods t and $t+1$, and two histories s^t and $s^{t+1} \in P(s^t)$ to obtain:

$$\frac{\beta u_c'(c_{t+1}(a_0, d_0, s^{t+1}), d_{t+1}'(a_0, d_0, s^{t+1}))\pi(s^{t+1}|s_0)}{u_c'(c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t))\pi(s^t|s_0)} = \frac{p_{t+1}(s^{t+1})}{p_t(s^t)} \frac{1 + \sum_{s^\tau \in P(s^t)} \mu(a_0, d_0, s^\tau)}{1 + \sum_{s^\tau \in P(s^{t+1})} \mu(a_0, d_0, s^\tau)}$$

Consider the agent whose participation constraint does not bind at history s^{t+1} following s^t. This would imply that $\mu(a_0, d_0, s^\tau) = 0$ for all $s^\tau \in P(s^t)$. Thus, the first order condition becomes the standard complete markets pricing relationship.

Now let's consider the prices generated from the planner's problem earlier where the participation constraints are not binding. Denote λ as the Lagrange multiplier associated with the promise keeping constraint. The first order condition on consumption can be combined with the first order condition on promised utility to obtain

$$qV_w' - \beta\lambda = 0$$

using the fact that $\lambda = \frac{1}{u_c'(c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t))}$ and the envelope condition $V_w = \lambda$, we derive

$$q = \beta \frac{u_c'(c_{t+1}(a_0, d_0, s^{t+1}), d_{t+1}'(a_0, d_0, s^{t+1}))}{u_c'(c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t))}$$

This gives us an condition on equilibrium prices that must satisfy

$$p_{t+1}(s^{t+1}) = \frac{qp_t(s^t)\pi(s^t|s_0)}{\pi(s^{t+1}|s_0)},$$

and from the normalization of $p_0 = 1$ and simple recursion we find

$$p_t(s^t) = q^t\pi(s^t|s_0). \tag{16}$$

The price, $p_t(s^t)$, of a contract that delivers a unit of goods, consumption or durables, contingent on shock history is composed of an aggregate intertemporal price q^t and a history dependent component equal to the probability that a history,s^t, occurs.

We can now turn our attention to the affordable allocations. Given prices solved above, the initial wealth level that makes the efficient allocation affordable for agent (w_0, d_0, s_0) is given by

$$\begin{aligned} a_0(w_0, d_0, s_0) &= c_0(w_0, d_0, s_0) - d_0 - y_0 + \sum_{t=1}^{\infty}\sum_{s^t|s_0} q^t\pi(s^t|s_0)(c_t(w_0, d_0, s^t) + \\ &\quad d_t'(w_0, d_0, s^t) - d_t(w_0, d_0, s^t) - y_t). \end{aligned} \tag{17}$$

Given that the optimal policy functions for consumption,c_t, and durables, d_t', are increasing functions in w_0 and d_0, we know that a_0 must also be an increasing function in w_0 and d_0. Therefore, this function is invertible and we denote the inverse as a_0^{-1}. We can thus denote the equilibrium allocations corresponding to the efficient allocation as

$$c_t(a_0, d_0, s^t) = c_t(a_0^{-1}(w_0, d_0, s_0), s^t), \tag{18}$$

and

$$d_t^{'}(a_0, d_0, s^t) = d_t'(a_0^{-1}(w_0, d_0, s_0), s^t) \tag{19}$$

for all t and histories, s^t. Our decentralization can be summarized by the following proposition which applies a theorem from Krueger and Perri (2001).

Proposition 11 *Suppose that* $\{c_t(w_0, d_0, s^t), d_t'(w_0, d_0, s^t)\}_{t=1}^{\infty}$ *is a stationary efficient allocation with associated price* $q > \beta$. *Then prices* $\{p_t(s^t)\}$ *and allocations* $\{c_t(a_0, d_0, s^t), d_t'(a_0, d_0, s^t)\}$, *as defined in (16), (18) and (19) are an equilibrium for initial distribution* Γ_0 *derived from* μ_0 *and (17).*

Proof. See Krueger and Perri (1999). ■

5 Risky Durable Goods and Exogenous Borrowing Constraints

In this section we describe an example exogenous incomplete market model of risky durables which we will later compare with the limited commitment model. Our proposed model basically extends the model in Aiyagari (1994) to include durable goods. We are going to analyze two versions of the exogenous borrowing constraint model. In the non-contingent version, the households have access to a non-contingent claim a', which pays one unit of consumption next period. In the durable contingent version, the households may buy a durable contingent claim a_i', which pays one unit of consumption if state i occurs next period. Households enter the period in one of 2 possible durable states and wealth level ϖ. Given this state, the household maximizes discounted lifetime utility by choosing an allocation consisting of consumption goods,c, durables goods,d', and assets, $\bar{a}$.

We formulate the household's problem recursively as

$$V(\varpi, s) = \max_{\bar{a}', d', c} u(c, d') + \beta V(\varpi', s')$$

where the household's current asset position is $\bar{a} = \{a\} \in A$ in the non-contingent version and $\bar{a} = \{a_1, a_2\} \in A$ in the durable contingent version, $\varpi \in W$ is the household's current period wealth, and $s \in S$ is the current state of the household. For the non-contingent version, in terms of current period wealth, households face the following budget constraint

$$c + d' + qa' = \varpi$$

where a' represents the household's choice of future assets and q represents the price of these assets. A similar constraint can be derived for the durable contingent version that takes the form

$$c + d' + q_1 a'_1 + q_2 a'_2 = \varpi$$

where a'_1 represents the claims that pay off if the household's durables remaining intact, a_2 represents the claims that pay off if the household suffers a catastrophic shock, and q_i represents the price of the corresponding contingent claim.

The wealth evolution equation for any household in terms of any future state, s', can be written as

$$\varpi'_{s'} = a' + y_{s'} + d'$$

where a' is the claim consumption good payoff for state s', $y_{s'}$ is the state specific endowment, and d' is the household's durable holdings which equals 0 if a catastrophic event occurs. Markets are incomplete via an exogenous borrowing constraint. This constraint sets a lower bound on household borrowing. The constraints take the following form

$$a' \geq B$$

where A represents the borrowing limit such that $B \leq 0$. We set the same limit for each security. Given the description of the environment, we can characterize the equilibrium in this modelled economy.

Definition 12 *A* ***recursive competitive equilibrium*** *for this economy consists of a collection of value functions* $V(\varpi, s)$*, decision rules* $d'(\varpi, s), c(\varpi, s),$ *and,* $\bar{a}'(\varpi, s)$*, price* q *(or prices* q_i*), and an invariant distribution* $\Gamma(\varpi, s)$ *such that*

(i) *Given* q*, the value function* V *and decision rules* d'*,* c*, and* $\bar{a}$ *solve the consumers' problem.*

(ii) *For all s, goods market clears:*

$$\int_W \sum_S \varpi \Gamma(d\varpi, s) = \int_W \sum_S \left[c(\varpi, s) + d'(\varpi, s)\right] \Gamma(d\varpi, s).$$

(iii) *The contingent claims market must clear for each s',*

$$\int_W \sum_{S'} a_i' \Gamma(d\varpi', s') = 0.$$

(iv) *Let T be an operator which maps the set of distributions into itself. Aggregation requires*

$$\Gamma'(\varpi', s') = T(\Gamma),$$

and T must be consistent with individual decisions.

6 Calibration,Baseline Results, and Empirical Facts

6.1 Calibration

Before we can numerically solve our model, a few parameters need to be calibrate. Specifically we need values for the utility parameter, θ, the probability of disaster, π_f, the endowment process, the household discount factor, β, and the exogenous borrowing limit, B. We calibrate the utility parameter θ so that the ratio of the aggregate durable stock to aggregate total consumption in the baseline model matches US data which we find to be about 1.28. We define the aggregate durable stock to be the sum of aggregate consumer durables plus aggregate residential fixed assets.[4] In terms of billions of 1996 US dollars we find

$$\frac{\text{Durable Stock}}{\text{GDP}} = \frac{2.9 + 8.9}{9.2} = 1.28.$$

In the baseline model this occurs when $\theta = 0.068$.

We obtain the probability of a catastrophic event from CEX data. We

[4]Data was obtained from the Bureau of Economic Analysis.

simply take the following ratio

$$\pi_f = \frac{\text{Number of Households Who Suffered a Disaster}}{\text{Total Households}}.$$

From CEX data we find that we find that a little over 1 million households in the US suffer from a catastrophic loss to their durables.[5] Thus, given that the CEX estimates a population of 108 million households in the US, we find $\pi_f \approx 1\%$. Later in the paper, we will vary this parameter to study the effects the changing risk level on efficient risk sharing.

The specification of the stochastic idiosyncratic labor productivity process is extremely important because of the implications that this choice has for the eventual distribution of wealth. Storesletten, Telmer and Yaron (2004) argue that the specification of labor income or productivity process for an individual household must allow for persistent and transitory components. Based on their empirical work from PSID data, we specify $\log(y_i)$ to be

$$\log(y_i) = \omega_i + \epsilon_i$$

$$\omega_i' = \rho\omega_i + v_i'$$

where $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ is the transitory component and ω_i is the persistent component. The innovation term associated with this component is assumed to $v_i \sim N(0, \sigma_v^2)$. They estimate $\rho = 0.935$, $\sigma_\epsilon^2 = 0.01$, and $\sigma_v^2 = 0.061$.

Using the methodology of Tauchen and Hussey (1991), we discretize this AR(1) process as a two state Markov chain. This generates the following results for the endowment levels, Y, and the endowment transition matrix,π_y,

$$Y = \{0.73, 1.27\}$$

[5]We define vehicles and houses as the major types of household durables. A household is said to have suffered a catastrophic loss to their durables if they replaced their car because it was stolen or damaged beyond repair, or if the expenditures on repairing their house exceed 50% of the value of the house.

$$\pi_y = \begin{bmatrix} 0.60 & 0.40 \\ 0.40 & 0.60 \end{bmatrix}.$$

The final calibration is on the preference parameter β which we set at 0.97 to generate $q = .975$ in the baseline model which is the same as roughly a 2.5% risk free rate. In the exogenous borrowing constraint model we use the same preference parameter, β, but alter the exogenous borrowing constraint to attain a real interest rate of 2.5% which occurs when $B = -0.735$.

6.2 Baseline Results

The calibrated version of the one-sided commitment model is solved numerically. Table 2 presents the baseline results. In addition, implied risk free rate is 2.5% , the coefficient of variation on consumption expenditures is 0.55, and the coefficient of variation on durable expenditures is 10.21. Noting that the coefficients of variations for perfect risk sharing are zero, we see that consumption is almost completely smoothed. Durables display substantially more dispersion. This difference in dispersion could be the direct affect of the catastrophic risk to durable goods. Later in the paper we vary the probability of disaster and examine this feature. When we compare these results to the exogenous borrowing constraint model, we find that the uncontingent exogenous constraint model yields larger coefficients of variation in consumption and durables of 9.72 and 28.17 respectfully. The durable contingent model also displays larger coefficients of variation in consumption and durables of 6.16 and 16.71 respectfully. The durable contingent model displays more risk sharing than the uncontingent model. However, both exogenous borrowing constraint models display substantially less risk sharing than the limited commitment model. This restates the previous notion that a model with limited commitment generates a higher degree of risk sharing than does an exogenous borrowing constraint model. Most of the differential is found relatively in non-durable consumption. The one-sided commitment model displays a dispersion 2.76 times tighter in durable expenditures and 17.7 time less dispersion in non-durable expenditures than the uncontingent exogenous borrowing constraint model. This difference in dispersion shrinks to 11.2 and 1.64 when we compare the durable contingent exogenous borrow constraint model with limited commitment.

Since we are going to compare these results with empirical data that have subsets which may have varying coefficients of variation in income, the final two columns display the ratios between the coefficients of variation in consumption and durables to income. The final two rows display the extreme results of autarky and perfect risk sharing for this environment.

Looking at the table we can notice that in fact there is substantial difference in the amount of risk sharing generated in the two models. Neither model is as volatile as the autarky solution meaning that both models display some risk sharing.

6.3 Empirical Facts

Given we have a measure of efficient risk-sharing, we can now check for similarity in empirical data. In this section we describe CEX household expenditure data. Specifically, we take a detailed look at spending on durables. Ideally we would prefer to have a data set which includes both the stock and flow of durables and identifies catastrophic shocks. However, since no data set exists that combines durable stocks, consumption, and income, we choose the next best option which is to look at durable expenditures.

We construct our data set from the 2000 Consumer Expenditure Survey. This survey provides information on the spending patterns of American households. It is currently the only survey which provides itemized expenditure patterns on both durable and nondurable goods. The itemized nature of the survey allows us to separate durable spending into two parts: replacement and expansion. Replacement expenditures include home, auto, and furniture repair and the replacement of a car because of a crash or theft. Expansion expenditures include other durable expenditures such as buying new houses and new appliances. Typical definitions of durable expenditures only include expansion expenditures, but since our model includes both types of durable expenditures, we include both types of expenditures in our empirical definition.

What does risk sharing look like in the cross section? Conditional on the aggregate state, perfect risk sharing would imply that differences in income should have no implication for expenditures. Thus the ratio of the dispersion of expenditures to the dispersion in income should be zero under perfect risk sharing. Over time the distribution of expenditures would collapse to a single point. Under autarky, we would expect large variations in consumption and durable expenditures as agents only have one insurance vehicle: their existing durable stock. Thus, the farther from risk sharing, the larger the ratio between income and expenditure dispersion.

All of our statistics are taken from the observations in the CEX data from the first quarter of 2000. Tables 3 and 4 present some basic statistics about household consumption and disposable income. The sample households are of working age and reported some positive labor income over the previous 12 months. We include only the individual who filed a complete income response. CEX data has a surprising large number of observations, about 20%, who did not report anything about their income. Thus, we are a little suspect about the income data in the CEX. We also remove any observations that report negative income or consumption.[6] The final data set consists of 4816 observations.

Household expenditures are presented as total expenditures, non-durable expenditures and durable expenditures. Non-durable expenditures includes expenditures on food, personal care, utilities, public transportation, fuel, and household operations. Durable expenditures include expenditures on housing, vehicles, appliances, household equipment, and repairs on durables. Total expenditures is the sum of non-durable expenditures, durable expenditures, and service expenditures. We also break the sample down according to the level of insurance premiums the household pays in any given period. The 410 households in the high insurance group reported paying at least $1000 in insurance premiums over the last three months. The 3604 observations which lie in the insurance group includes any household that paid any insurance premiums. The final 1216 households in the no insurance

[6]For a more detailed discussion on the limits of CEX income data see Nelson (1994).

groups reported no expenditures on insurance premiums. In this way, we can analyze the risk-sharing effects of real-world insurance coverage.

We then use the data from the previous two tables to construct coefficients of variation for disposable income, total household expenditures, non-durable expenditures and durable expenditures. These statistics are presented in Table 5. The use of coefficient of variation lies partly in the fact that the mean and standard deviation of income tend to change together. The coefficient of variation allows us to control for level differences in income and expenditures across groups, generating a clean look at income and expenditure dispersion.

To control for the fact that the coefficient of variation in disposable income varies across groups, we construct ratios which compare the dispersion of income with expenditures. This technique allows us to compare the dispersion in the data with that found in the model.

These ratios are presented in Table 6. We see that durable expenditures are highly volatile as compared to non-durables. In fact, the relative dispersion of durable expenditures to income is about 1.4 to 1. On average the dispersion of non-durables to income is roughly two-thirds of disposable income. An interesting feature is that the level of household insurance premium expenditures has little effect on the dispersion of expenditures. It appears that current insurance arrangements are not an effective expenditure smoother.

It is obvious that the data displays much more dispersion than what is found in either the one-sided commitment or the exogenous borrowing constraint models. We find that the ratio of coefficients of variation are substantially smaller in the models versus the data. The ratio for consumption expenditures is 0.648 larger in the data than it is in the baseline one-sided commitment model. The ratio for durable expenditures is 1.248 larger in the data. The exogenous borrowing constraint model does somewhat better with these ratio differences being 0.406 and 0.906 respectfully. Even taking into account that a two state income process biases the model towards higher risk sharing, it is clear that limited commitment captures only part of the incomplete risk sharing behavior we find in the data. Some additional mechanisms must be present that further inhibit risk sharing.

6.4 Changing the Probability of Disaster

In this section, we study the results of the limited commitment model with different probabilities of disasters. The section has two objectives. First, we are interested in documenting how efficient risk sharing changes as we alter the probability of disaster. Are the results of the baseline robust to changes in the probability of catastrophic loss? Second, we are interested in addressing some applications of this model. The risk to durables is not constant across any particular region. The probability of a loss resulting from a hurricane is much greater in Florida than it is in Utah. This has implications for efficient risk sharing arrangements between and across different regions. We can begin to address these issues in this section. For comparison we run the model with catastrophic probabilities varying from 0.1%, and 2%. Table 7 presents the results from this numerical experiment.

As the probability of disaster increases, there is an increase in dispersion of durable expenditures. A movement from 0.1% to 2.0% disaster probability nearly doubles the dispersion in durable expenditures. When comparing across states, the model predicts that there should be little variation in the dispersion of non-durable expenditures. If anything, it predicts a modest decrease in the dispersion in non-durables as the probability of disaster increases. This is mostly due to agents switching out of durable goods, and into non-durables thus make non-durable consumption slightly more exposed to income risk. So, this model would predict that states with higher probabilities of disasters should have a larger dispersion in durable expenditures. Thus, Florida would have higher durable dispersion than a state like Utah where the probability of disaster is much lower. It also predicts that the dispersion of non-durables is slightly inversely related to the probability of disaster. Table 8 presents some evidence on this feature. Table 8 displays the dispersion of durables for some of the most populous states in the US. We have broken the states into high and low risk states which corresponds to the probability that a household in the state may experience a catastrophe such as a hurricane, tornado, earthquake, or blizzard.

Durables exhibit a larger dispersion in the states which would have a higher probability of disaster. We also see that, except for Utah, the dispersion of non-durables is slightly higher in the states where the probability of disaster is lower. Given we have a correct proxy for the probability of disaster, the model's predictions about dispersion and probability of disaster are correct qualitatively. Much more empirical work needs to be done for a complete analysis of this topic.

For a more robust test, we obtained data from the hazard research lab at the University of South Carolina. This data estimates state wide risk levels associated with natural disasters and other catastrophic events[7]. Looking at data from 1975 to 1998, they score each state in one of three hazard categories: economic loss, deaths, and number of events. These three categories are then averaged, and each state is given an overall hazard rate. States with a high hazard rate are considered riskier than states with low hazard rates. For a further empirical test of the previous result, we ran correlations between state level expenditure dispersion and hazard rates. The results are presented in Table 9. The second column presents the correlation between the ratio of the coefficient of variation in non-durable expenditures to the coefficient of variation in income and different hazard rates where hazard score represents the overall rate of hazard. We find a negative correlation between hazard rates and our measure of non-durable dispersion. If we use the hazard score as a measure of the probability of disaster, we find that our results are validated. Riskier states have lower dispersion in non-durable expenditures. When looking at durables in the third column, we find the opposite relationship. There is a positive correlation between hazard rates and durable dispersion. Thus riskier states will tend to have greater dispersion in durable expenditures which is predicted by the model.

[7]These events include natural disasters such as floods, hail, tornadoes, wind storms, lightning strikes, wildfires, drought, winter storms, hurricanes, earthquakes, and volcanic eruptions, plus man made disasters such as chemical spills, nuclear power plant accidents, and other toxic releases.

7 Conclusions

In conclusion, the efficient risk sharing arrangement displays substantial though not perfect risk sharing. The introduction of endogenous autarky with risky durable goods does not change the typical results of a one-sided commitment model. To summarize the results of the baseline model:

1. One-sided commitment captures only a portion of the incomplete risk sharing we find CEX expenditure data.
2. The one-sided commitment model displays substantially more risk sharing than the exogenous borrowing constrained model.
3. The one-sided commitment models finds that the efficient allocation displays substantial though not perfect risk sharing.
4. The introduction of durable goods induces substantial consumption smoothing even in autarky.

One obvious shortcoming of the current model is its inability to generate sufficient dispersion in both non-durable and durable expenditures. This would imply that something else in addition to one-sided commitment is hindering risk sharing. One possibility is an issue of incentive compatibility or incomplete information. Consider an environment of one-sided commitment, but include the fact that endowments are private information. In such a world, there could be an incentive for certain agents to lie about their current endowment state in hopes of receiving a larger transfer from the planner. Given that the planner does not want this happen, the planner needs to enforce an incentive compatibility constraint where all agents receive higher utility from reporting their true endowment state. The introduction of this type of constraint into the current environment would clearly decrease the efficient level of risk-sharing by contracting the region of feasible contracts. Other possible deterrents could be moral hazard or adverse selection problems which could be introduced into this environment. In this case, we would need to endogenize the probability of disaster. We could address some of the issues surrounding the health and life insurance industries.

We also find that variations in the probability of catastrophic loss have relatively large effects on the dispersion of durable expenditures. As the probability of disaster increases, the efficient amount of durable expenditure dispersion increase. We find a weaker and reversed result for non-durables. As the probability of disaster increases, the efficient dispersion in non-durables decreases. Future empirical tests could check on this phenomena across states and countries. Our basic empirical analysis seems to support such behavior, but a more complete empirical examination of catastrophic risk is needed before addressing these issues further. Given such an analysis, we could apply the model across regions that have variable disaster probabilities. This would allow us to begin looking at other interesting topics. We could study the efficient risk sharing characteristics of an intermediary who must optimally price a contract across two groups with different disaster probabilities.

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8 Appendix

8.1 Theoretical Proofs

Proof of Proposition 1:

We begin by showing that the value function under autarky is increasing and concave. Assume that d_t and $\overline{d}_t$ are two initial durable states such that $\overline{d}_t > d_t$ Then for the agent with d_t we know

$$V(d_t, s_t) \geq \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, d'_j) = u(c_t, d'_t) + \sum_{j=t+1}^{\infty} \beta^{j-t} u(c_j, d'_j) \tag{A1}$$

Now assume we have an agent that has $\overline{d}_t$. Since the agent can now afford additional consumption and durable purchases in period t, denoted $\overline{c}_t$ and $\overline{d}'_t$, and still consume the allocations c_j and d'_j in every subsequent period, we must have

$$V(\overline{d}_t, s_t) \geq u(\overline{c}_t, \overline{d}'_t) + \sum_{j=t+1}^{\infty} \beta^{j-t} u(c_j, d'_j). \tag{A2}$$

By the monotonicity of $u(c_t, d'_t)$ we must have

$$V(\overline{d}_t, s_t) > V(d_t, s_t) \tag{A3}$$

We now need to show that the choice set under autarky is a convex set. The budget set under autarky is written as

$$c_t + d'_t \leq y_t + d_t \tag{A4}$$

The choice set (c_t, d'_t) is convex if for two feasible choices (c_t, d'_t) and $(\overline{c}_t, \overline{d}'_t)$ then $(\alpha c_t + (1-\alpha)\overline{c}_t, \alpha d'_t + (1-\alpha)\overline{d}'_t)$ is also feasible. We see that

$$\begin{aligned}
& \alpha c_t + (1-\alpha)\overline{c}_t + \alpha d'_t + (1-\alpha)\overline{d}'_t \\
= \; & \alpha(c_t + d'_t) + (1-\alpha)(\overline{c}_t + \overline{d}'_t) \\
\leq \; & \alpha(y_t + d_t) + (1-\alpha)(y_t + d_t) \\
= \; & y_t + d_t
\end{aligned}$$

thus the choice set is a convex set and the cross-product of these sets form a convex set. Given this, the value function must satisfy

$$
\begin{aligned}
V(\alpha d_t + (1-\alpha)\overline{d}_t, s_t) &\geq \sum_{j=t}^{\infty} \beta^{j-t} u(\alpha c_j + (1-\alpha)\overline{c}_j, \alpha d'_j + (1-\alpha)\overline{d}'_j) \\
&\geq \alpha \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, d'_j) + (1-\alpha) \sum_{j=t}^{\infty} \beta^{j-t} u(\overline{c}_j, \overline{d}'_j) \\
&= \alpha V(d_t, s_t) + (1-\alpha) V(\overline{d_t}, s_t); \qquad (20)
\end{aligned}
$$

thus the value function is concave with respect to d_t.

We can now prove that consumption and durable purchases are increasing in d_t. Assume $d_t < \overline{d}_t$. We know that

$$
V(\overline{d}_{t+1}, s_{t+1}) > V(d_{t+1}, s_{t+1}) \qquad (21)
$$

However, we also know that $V(d_t, s_t)$ is strictly concave, so it must be the case that

$$
V(\alpha\overline{d}_{t+1} + (1-\alpha)d_{t+1}, s_{t+1}) > \alpha V(\overline{d}_{t+1}, s_{t+1}) + (1-\alpha)V(d_{t+1}, s_{t+1}) \quad (22)
$$

But $\alpha\overline{d}_{t+1} + (1-\alpha)d_{t+1}$ is feasible whenever d_t and $\overline{d}_t$ are feasible, so we can choose it. Since $\alpha \in (0,1)$, it must be the case that increasing durable purchases by less than the difference between d_t and $\overline{d}_t$ increases lifetime utility. Since the budget constraint must hold with equality, both consumption and durable purchases must rise with d_t.

From the form of the utility function we know $d'_t > 0$ and $c_t > 0$ when $d_t = 0$. Thus at the lower bound $d'_t > d_t$.Since both consumption and durable purchases are increasing with respect to the current durable state, we know that durable purchases have a slope less than one. Thus there must exist some d_U such that $d_t > d'_t$. Since we know that d_t is increasing faster than d'_t,we know that for all $d_t \geq d_U$, $d'_t < d_t$. **QED**

Proof of Lemma 1:

For every $(w, d, s) \in \Omega$ the objective function is continuous in c, d', and w' and the constraint set is compact and non-empty. Therefore, a minimum must exist. The value function and $C(\Omega)$ are bounded by $h_L(w, d, s) \leq h \leq h_U(w, d, s)$. It follows that the operator $T_q V$ is a bounded function. From the Theorem of the maximum and continuity of the constraint set, we find that $T_q V$ is continuous. Therefore T_q forms a map $C(\Omega) \to C(\Omega)$. We simply need to check whether Blackwell's theorem holds for this operator.

Suppose $v(w, d, s) \leq w(w, d, s)$ for all w, d, and s. Thus, the objective function for which T_w is maximized is uniformly higher than the function for T_v.Thus, the operator is clearly monotone. We also see that for some constant a,

$$\begin{aligned}
T_q(V+a)(w,d,s) &= \min_{c_s,d'_s,w_s^1,w_s^2,w_s^3,w_s^4} \left[\begin{array}{c} c_s + d_s^{'} - d + \\ q \left[\begin{array}{c} (1-\pi_f)\sum_{y'} \pi(y'|y)V(w_{s,s'}, d'_s, y') + a \\ +\pi_f \sum_{y'} \pi(y'|y)V(w_{s,s'}, 0, y') + a \end{array} \right] \end{array} \right] \\
&= \min_{c_s,d'_s,w_s^1,w_s^2,w_s^3,w_s^4} \left[\begin{array}{c} c_s + d_s^{'} - d + \\ q \left[\begin{array}{c} (1-\pi_f)\sum_{y'} \pi(y'|y)V(w_{s,s'}, d'_s, y') \\ +\pi_f \sum_{y'} \pi(y'|y)V(w_{s,s'}, 0, y') \end{array} \right] + qa \end{array} \right] \\
&= T_q(V)(w,d,s) + qa
\end{aligned}$$

Thus as long as $q < 1$, the operator satisfies discounting, and the Blackwell theorem holds. The operator is a contraction with modulus q. **QED**

Proof of Lemma 2:

We begin by showing that the value function is increasing with respect to w. Assume that w and $\overline{w}$ are two initial utility states such that $\overline{w} > w$. We then need to show $(T_q V(w,d,s)) < T_q V(\overline{w}, d, s)$. Let $\overline{c}, \overline{w}', \overline{d}'$ be the optimal choices for $(\overline{w}, d, s)$. There exists choices $w' = \overline{w}'$, $d' = \overline{d}'$ and $c < \overline{c}$ that are feasible for (w, d, s). Therefore,

$$\begin{aligned}
T_q V(\overline{w}, d, s) &= \overline{c} + \overline{d}' - d + q \sum_{s' \in S} \pi(s') V_q(\overline{w}', \overline{d}', s') \\
&> c + d' - d + q \sum_{s' \in S} \pi(s') V_q(w', d', s') \\
&= T_q V(w, d, s)
\end{aligned}$$

Thus the value function is increasing in w.

For decreasing in d, assume that d and $\overline{d}$ are two initial states such that $\overline{d} > d$. Then we need to show $(T_q V(w, \overline{d}, s)) < T_q V(w, d, s)$. Let $\overline{c}$, $\overline{w}'$, $\overline{d}'$ be the optimal choices for $(w, \overline{d}, s)$. There exists choices $w' = \overline{w}'$, $d' = \overline{d}'$, and

$c = \overline{c}$ that are feasible for (w, d, s). Therefore,

$$\begin{aligned} T_qV(w, \overline{d}, s) &= \overline{c} + \overline{d}' - \overline{d} + q \sum_{s' \in S} \pi(s') V_q(\overline{w}', \overline{d}', s') \\ &= c + d' - \overline{d} + q \sum_{s' \in S} \pi(s') V_q(w', d', s') \\ &< c + d' - d + q \sum_{s' \in S} \pi(s') V_q(w', d', s') \\ &= T_qV(w, d, s) \end{aligned}$$

Thus the function is decreasing with respect to d.

Finally, we show that the value function is convex. Define

$$\begin{aligned} w_\lambda &= \lambda w + (1-\lambda)\,\overline{w} \\ w'_\lambda &= \lambda w' + (1-\lambda)\,\overline{w}' \\ c_\lambda &= \lambda c + (1-\lambda)\,\overline{c} \\ d_\lambda &= \lambda d + (1-\lambda)\,\overline{d} \\ d'_\lambda &= \lambda d' + (1-\lambda)\,\overline{d}'. \end{aligned}$$

Since these choices are feasible for $(w_\lambda, d_\lambda, s)$ and we have

$$\begin{aligned} (T_qV)(w_\lambda, d_\lambda, s) &\leq c_\lambda + d'_\lambda - d_\lambda + q \sum_{s' \in S} \pi\left(s'\right) V_q\left(w'_\lambda, d'_\lambda, s'\right) \\ &\leq \lambda \left[c + d' - d + q \sum_{s' \in S} \pi\left(s'\right) V_q\left(w', d', s'\right) \right] + \\ &\quad (1-\lambda) \left[\overline{c} + \overline{d}' - \overline{d} + q \sum_{s' \in S} \pi\left(s'\right) V_q\left(\overline{w}', \overline{d}', s'\right) \right] \\ &= \lambda (T_qV)(w, d, s) + (1-\lambda) V_q\left(\overline{w}, \overline{d}, s\right) \end{aligned}$$

we have convexity of (T_qV) whenever V_q is convex. Given that the set of convex functions is closed in the space of continuous functions, the fixed point V must be convex as well. **QED**

Proof of Lemma 4:

Suppose the participation constraint is binding, $w' = V^{AUT}(d', s')$. From the previous result we know that since d' is increasing in w and V^{AUT} is increasing in d', then w'must be increasing in w. Now suppose that the participation constraint is not binding, $w' > V^{AUT}(d', s')$, the previous

Lemma showed that $w' = w$. Thus the promise utilities are increasing in w.
QED

Proof of Lemma 5:

We know $w' > V^{AUT}(\overline{d}, s')$ by assumption. Combining the first order condition on promise utility with the envelope condition on w. It is easy to find $\frac{\beta V_w(w,d,s)}{q} = V_{w'}(w', d', s')$. Since, $q > \beta$, we have $V_w(w, d, s) < V_{w'}(w', d', s')$ and thus if V is convex, $w' < w$. Thus the policy function lies strictly under the 45° degree line whenever the function is increasing. Since, participation required $w' \geq V^{AUT}(d', s')$ for all w. Whenever $w < V^{AUT}(d, s)$,it must be the case that $w' = V^{AUT}(d', s') > w$. Thus for low levels of w, the policy lies strictly above the 45°. Given that there exists a choice w' for every w, and since w' is continuous, we know there exists a point such that when $w = V^{AUT}(d, s)$, $w' = V^{AUT}(d', s')$. From the first result we have, $w' < w$, for all $w > V^{AUT}(d', s')$. **QED**

Proof of Proposition 2:

Take $\overline{w} = \max_y U^{AUT}(\overline{d}, s) + \epsilon$, for $\epsilon > 0$, where $\overline{d}$ is the upper bound on durables derived from Proposition 1. If $w' > V^{AUT}(\overline{d}, s')$, then the previous Lemma yields the result. If $w' = V^{AUT}(\overline{d}, s')$, then $w' = V^{AUT}(\overline{d}, s') \leq \max_y U^{AUT}(\overline{d}, s) < \overline{w}$. **QED**

8.2 Computations

In this section we briefly describe the computational method used to solve the one-sided commitment model. We discretize the state space by choosing a finite grid over utilities and durable holdings.

- To compute the model.

  1. Pick a polynomial to approximate the value function V.
  2. Guess an initial set of coefficients Φ for this polynomial.
  3. Guess an initial q and invariant distribution Γ.
  4. Solve the autarky problem to obtain participation constraints.
  5. Iterate over the state space to obtain policy functions c, d', and promised utilities and the updated value function V'. We use interpolation between grid points thus making choices continuous.
  6. Use OLS to obtain a new set of coefficients Φ'. If $\Phi = \Phi'$ we have convergence. If not, return to step 5. Check the fit of the regression. If sufficient ($R^2 > 0.99$) continue, else return to step 1 with a higher order polynomial.

7. Use decision rules to compute new Γ, aggregate consumption C , and aggregate durable investment $D' - D$.
8. Check for market clearing. If goods market clears we have converged, else update q using Brent's method and return to step 3.

Table 1
Ten Most Costly Catastrophic Events in the US[†]

Date	Event	Cost in millions 2000 US$
Aug. 2005	Hurricane Katrina	$40,600
Sep. 2001	World Trade Center	$35,600
Aug. 1992	Hurricane Andrew	$21,600
Jan. 1994	Northridge Earthquake	$16,472
Oct. 2005	Hurricane Wilma	$10,300
Aug. 2004	Hurricane Charley	$7,700
Sep. 2004	Hurricane Ivan	$7,400
Sep. 1989	Hurricane Hugo	$6,600
Sep. 2005	Hurricane Rita	$5,000
Aug. 2004	Hurricane Frances	$4,800

[†]The Fact Book 2002, Insurance Information Institute, 2002.

Table 2
Baseline Results

Model	q	B	$CV\ C$	$CV\ D$	$\frac{CV(C)}{CV(INCOME)}$	$\frac{CV(D)}{CV(INCOME)}$
Limited Commitment	.975		0.55	10.21	0.012	0.202
Exo.-Uncontingent	.975	−0.735	9.72	28.17	0.193	0.558
Exo. Durable Contingent	.975	−0.947	6.16	16.71	0.122	0.331
Autarky	.985	−0.000	10.88	38.94	0.216	0.771
Perfect Risk Sharing	.970	−23.60	0.00	0.00	0.000	0.000

Table 3
Standard Deviations

	Disp. Income†	Total Exp.	Non-Durables	Durables
Total Sample	$40,696	$8,146	$1,661	$6,412
Pays High Insurance	$64,908	$12,899	$2,361	$10,062
Pays Insurance	$40,795	$8,318	$1,645	$5,436
No Insurance Payments	$35,955	$6,448	$1,505	$5,655

†Income data is presented in annualized terms.

Expenditure data is presented as quarterly measures.

The relative size of the standard deviations to average are similar to those found in Mace (1991).

Table 4
Averages

	Disp. Income	Total Exp.	Non-Durables	Durables
Total Sample	$45,115	$9,707	$2,774	$4,896
Pays High Insurance	$86,287	$18,964	$4,407	$9,614
Pays Insurance	$50,189	$10,804	$2,999	$5,436
No Insurance Payments	$29,092	$6,245	$2,065	$3,191

Table 5
Coefficients of Variation

	Disp. Income	Total Exp.	Non-Durables	Durables
Total Sample	90.2	83.9	59.9	131.0
Pays High Insurance	75.2	68.0	53.6	104.6
Pays Insurance	81.3	77.0	54.9	120.3
No Insurance Payments	123.6	103.2	72.9	177.2

Table 6

Ratio of Coefficients of Variation

	Total Exp.	Non-Durables	Durables
Total Sample	0.93	0.66	1.45
High Insurance	0.90	0.71	1.39
Pays Insurance	0.95	0.68	1.48
No Insurance Payments	0.83	0.59	1.43

Table 7

Varying Catastrophic Probability

Disaster Prob.	q	$CV\ CON$	$CV\ DURABLES$	$\frac{CV(C)}{CV(INCOME)}$	$\frac{CV(D)}{CV(INCOME)}$
0.1%	0.998	0.91	6.46	0.018	0.128
0.5%	0.987	0.74	7.83	0.015	0.155
1.0%	0.975	0.55	10.21	0.011	0.202
2.0%	0.972	0.58	14.21	0.011	0.281

Table 8

Ratio of Coefficients of Variation by State

State	Tot Exp.	Non Durables	Durables
High Risk:			
Florida	1.018	0.600	1.498
Texas	0.920	0.600	1.416
Low Risk:			
New Jersey	0.887	0.744	1.248
Pennsylvania	1.033	0.899	1.093
Utah	0.670	0.664	0.854

Table 9

Correlations between Ration of CV and Hazard Rates

Measure	Non-Durables	Durables
Hazard Score	−0.277	0.209
Damage	−0.229	0.125
Death	−0.283	0.294
Event	−0.195	0.035

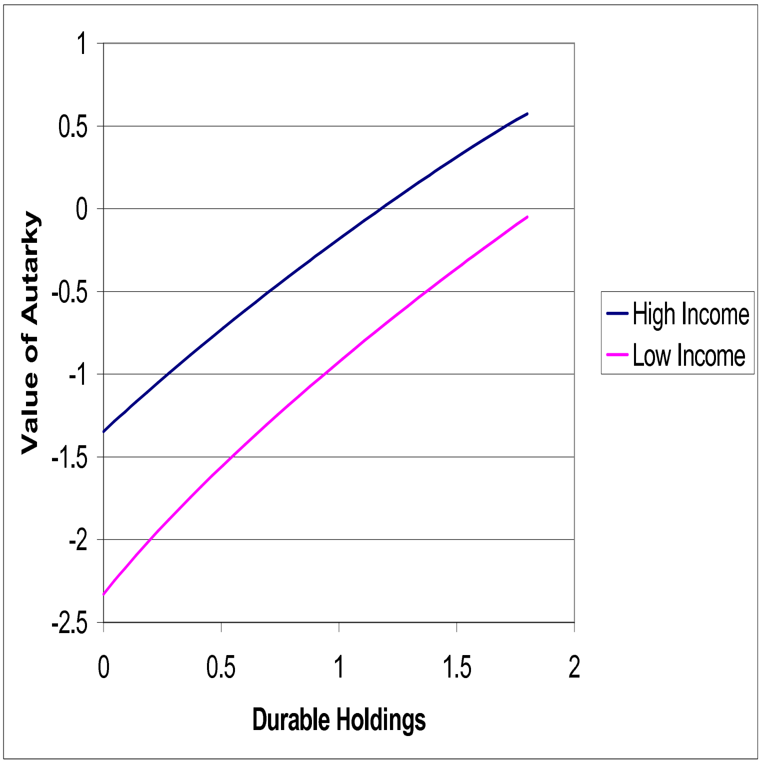

Figure 1: Value of Autarky

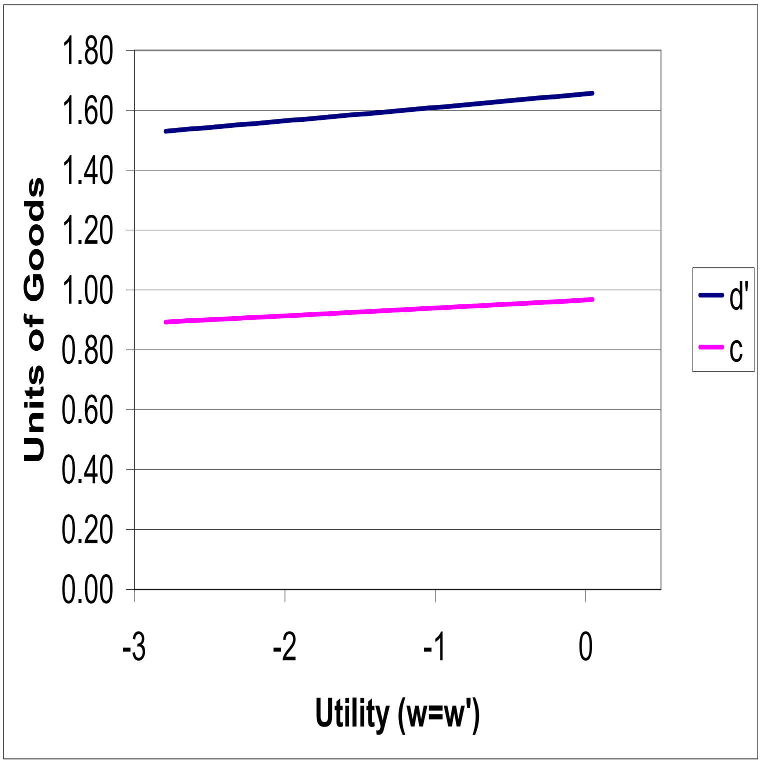

Figure 2: Perfect Risk Sharing Allocations

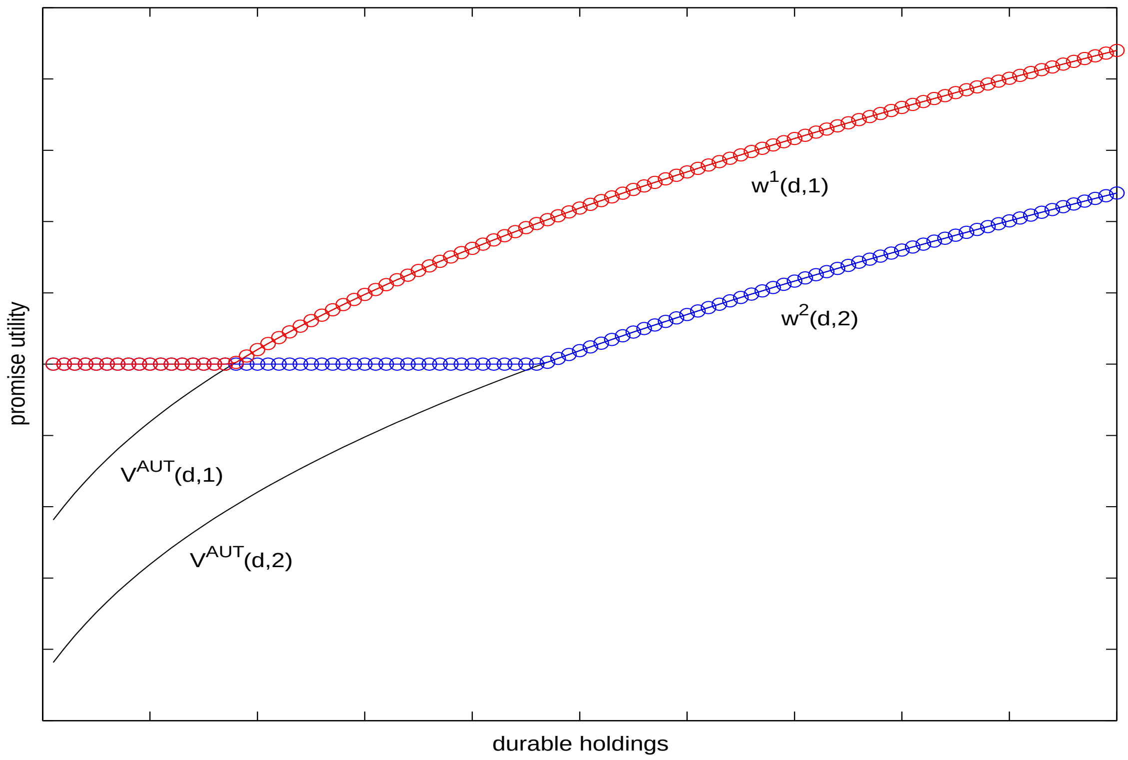

Figure 3: Promise Utility as a Function of Durable Holdings

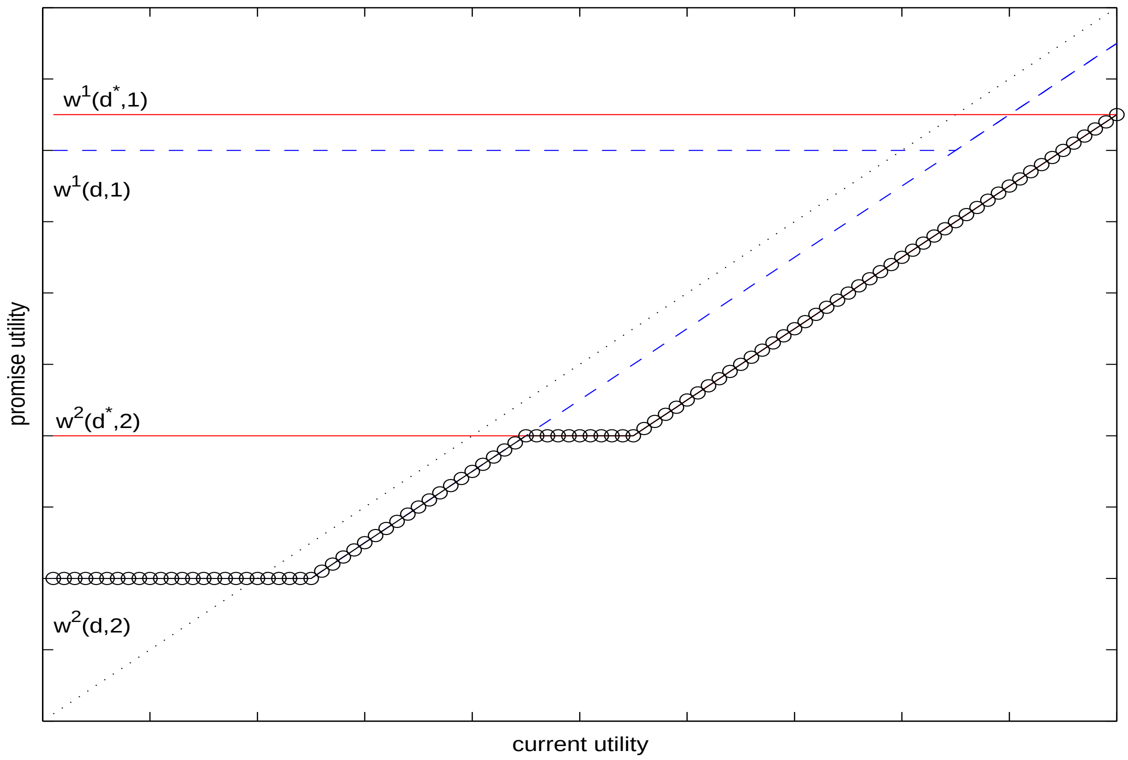

Figure 4: Promise Utility as a Function of Current Utility

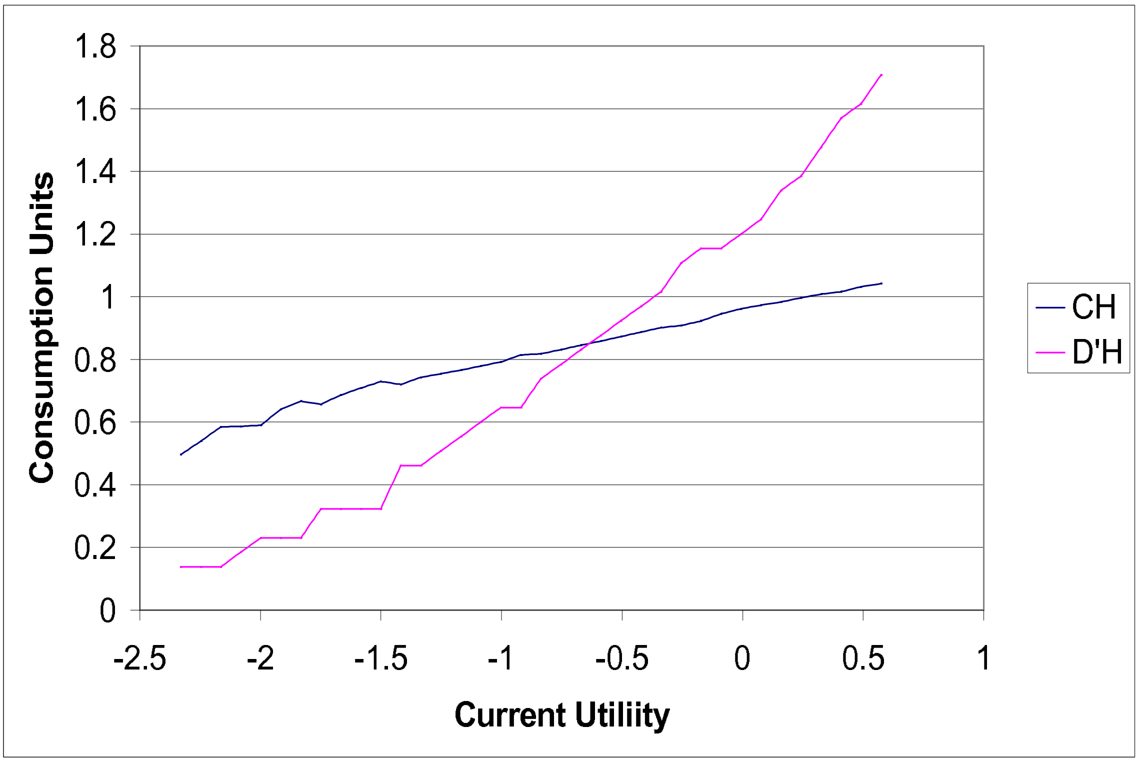

Figure 5: Allocations as a Function of Current Utility