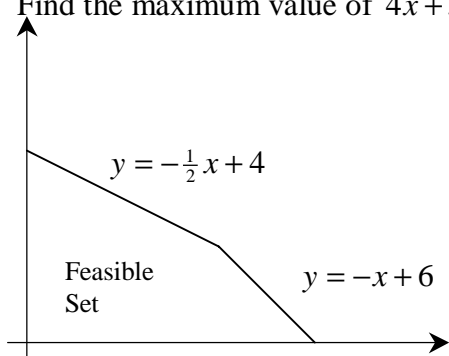


Midterm Examination #2
Math 111
Algebra for Applications
Wednesday, November 4, 1998

Each problem is 10 points, unless noted otherwise. Calculators are permitted, but all work must be shown.

§1 Computation:

1. Graph the feasible set for the system of inequalities $\begin{cases} 6x + 3y \leq 96, \\ x + y \leq 18, \\ 2x + 6y \leq 72, \\ x \geq 0, y \geq 0. \end{cases}$
2. Find the maximum value of $4x + 5y$ on the given feasible set.



3. Write the matrix corresponding to the following linear programming problem

$$\text{Maximize } x + 2y + z \text{ subject to the constraints } \begin{cases} x - y + 2z \leq 10, \\ 2x + y + 3z \leq 12, \\ x \geq 0, y \geq 0, z \geq 0. \end{cases}$$

4. While using the simplex method to solve a linear programming problem, you obtain the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 10 \\ 3 & 4 & 0 & 1 & 0 & 6 \\ -10 & -1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Choose the next pivot element, and pivot the matrix about that element.

§2 Comprehension:

5. What is the fundamental theorem of linear programming? Explain its importance.

§3 Applications:

6. Give an algebraic formulation of the following problem, and convert it to an appropriate maximization problem. *DO NOT SOLVE IT!*

A citizen decides to campaign for the election of a candidate for city council. Her goal is to generate at least 200 votes by a combination of door-to-door canvassing, letter writing, and phone calls. She figures that each hour of door-to-door canvassing will generate four votes, each hour of letter writing will generate three votes, and each hour of phone calls will generate two votes. She would like to devote at least seven hours to phone calls, and spend at most half her time at door-to-door canvassing. How much time should she allocate to each task to achieve her goal in the last amount of time?

7. (20 points) Solve the following problem using graphical methods.

A farmer has 100 acres on which to plant oats or corn. Each acre of oats requires \$18 capital and two hours of labor. Each acre of corn requires \$36 capital and six hours of labor. Labor costs are \$8 per hour. The farmer has \$2100 available for capital and \$2400 available for labor. If the revenue is \$55 from each acre of oats, and \$125 for each acre of corn, what planting combination will yield the greatest total profit? (Here profit is revenue plus leftover capital and cash reserve.) What is the maximum profit?

8. (20 points) Solve the following problem using the simplex method.

A furniture manufacturer makes two types of furniture- chairs and sofas. For simplicity, divide the production process into three distinct operations- carpentry, finishing, and upholstery. The amount of labor for each operation varies. Manufacture of a chair requires 6 hours of carpentry, 1 hour of finishing, and 2 hours of upholstery. Manufacture of a sofa requires 3 hours of carpentry, 1 hour of finishing, and 6 hours of upholstery. Each day the factory has 96 labor-hours available for carpentry, 18 labor-hours for finishing, and 72-labor hours for upholstery. The profit per chair is \$80 and the profit per sofa is \$70. How many sofas and chairs should be made to maximize the profit?