

Final Examination #1
Math 111
Algebra for Applications
Wednesday, December 16, 1998

This exam has 200 points. Unless otherwise indicated, each problem is worth 10 points.

§1 Computation:

1) Find the equation of the line passing through the point $(5,6)$ with slope $\frac{3}{5}$. What is the y-intercept of this line?

2) Graph the feasible set for the inequalities $\begin{cases} x + 2y \geq 2 \\ 3x - y \geq 3 \end{cases}$.

3) Determine all solutions of the system of equations $\begin{cases} 2x + 2y + 4z = 8 \\ x - y + 2z = 2 \\ -x + 5y - 2z = 2 \end{cases}$.

4) Calculate the following products, if they are defined.

a) $\begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & -4 \\ 2 & -1 \end{bmatrix}$.

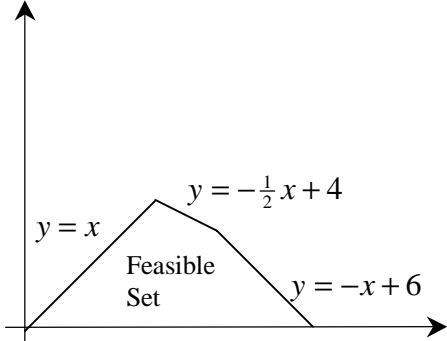
b) $\begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$.

5) Write the following system of equations in the form $AX = B$ for matrices A , X , and B . Solve the system by calculating the inverse of A and performing a matrix multiplication.

$$\begin{cases} -2x + 4y = 2 \\ -3x + 7y = 7 \end{cases}$$

6) Compute the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{bmatrix}$.

7) Find the maximum value of $4x + 5y$ on the given feasible set.



- 8) Write the simplex tableau corresponding to the following problem. Maximize $P = 80x + 70y$ subject to the following conditions.

$$\begin{cases} 6x + 3y \leq 96 \\ x + y \leq 18 \\ 2x + 6y \leq 72 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

- 9) Draw a three circle Venn diagram corresponding to the set $R \cap S \cap T'$.
- 10) Which of the following probabilities are feasible for an experiment having sample space $\{s_1, s_2, s_3\}$?
- $\Pr(s_1) = 0.4$, $\Pr(s_2) = 0.4$, and $\Pr(s_3) = 0.4$.
 - $\Pr(s_1) = 0.5$, $\Pr(s_2) = 0.7$, and $\Pr(s_3) = -0.2$.
 - $\Pr(s_1) = 2$, $\Pr(s_2) = 1$, and $\Pr(s_3) = 0.5$.
 - $\Pr(s_1) = \frac{1}{4}$, $\Pr(s_2) = \frac{1}{2}$, and $\Pr(s_3) = \frac{1}{4}$.

§2 Comprehension:

- What are the elementary row operations for a system of equations?
- What is the fundamental theorem of linear programming?
- Explain the meaning of the following terms; include an example: experiment, outcome, sample space, event.
- What is the inclusion-exclusion principle?
- What is the binomial theorem? Explain why it is true.

§3 Applications:

- 16) The amount of tropical rainforest in Central America has been decreasing steadily in recent years. The amount y in thousands of square miles x years after 1969 is estimated by the linear equation

$$y = \left(-\frac{25}{8}\right)x + 130.$$

- Sketch the graph of this linear equation.
 - What interpretation can be given to the y -intercept of this graph?
 - When were there 80,000 square miles of tropical rain forest?
 - If this trend continues, how large will the rain forest be in 2001?
- 17) (20 points) Solve the following problem using the simplex method.
- An appliance store sells three brands of color TV sets, brands A, B, and C. The profit per set is \$30 for brand A, \$50 for brand B, and \$60 for brand C. The total warehouse space allotted to all brands is sufficient for 600 sets, and the inventory is delivered once per month. At least 100 customers per month will demand brand A, at least 50 will demand brand B, and at least 200 will

demand either brand B or brand C. How can the appliance store satisfy all of these constraints and earn maximum profit?

- 18) A restaurant lists 6 appetizers, 10 entrées, and 5 desserts. How many ways can a diner select a three course meal?
- 19) An exam contains five “true or false” questions. What is the probability that a student, guessing randomly, will get three or more right?