

Exam #2
Math 111
March 18, 2005

Name _____

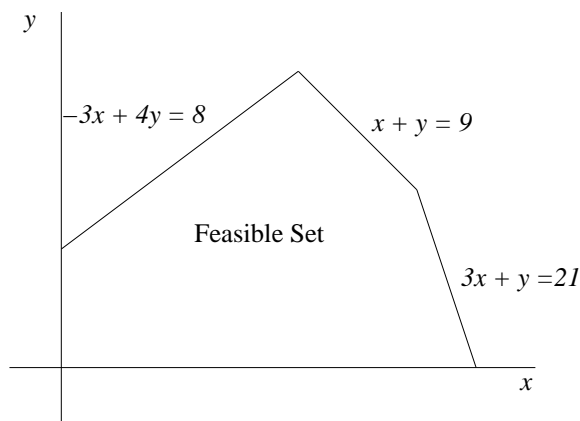
All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Graph the feasible set for the system of inequalities

$$\begin{cases} 2x + y \leq 8, \\ x + 2y \leq 6, \\ x, y \geq 0. \end{cases}$$

2. Find the maximum of $f(x) = 3x + 11y$ on the feasible set



3. Write the simplex tableau corresponding to the following problem:
Maximize $M = x + 5y$ subject to the constraints

$$\begin{cases} x + y \leq 20, \\ -x - 2y \geq -25, \\ x, y \geq 0. \end{cases}$$

4. Consider the simplex tableau for the variables $x, y, u, v,$ and M

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 & 0 & 4 \\ 0 & 5 & 2 & 0 & 1 & 0 \end{bmatrix}.$$

Find the Group I variables and the Group II variables. Find a choice of the variables $x, y, u, v,$ and M that solves the system of equations represented by the tableaux.

5. Consider the simplex tableau

$$\begin{bmatrix} 2 & 4 & 1 & 0 & 0 & 6 \\ 3 & 1 & 0 & 1 & 0 & 1 \\ -1 & -2 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Determine the appropriate pivot element, and pivot about that element.

6. Use the simplex method to maximize the function $10x + y$ subject to the constraints

$$\begin{cases} x + 2y \leq 10, \\ 3x + 4y \leq 6 \\ x, y \geq 0. \end{cases}$$

§2 Comprehension

7. What is the Condition for a Maximum in the simplex method?

§3 Application

8. Consider the following problem:

A pension fund has \$30 million to invest. The money is to be divided among Treasury notes, corporate bonds, and stocks. The rules for administration of this fund require that at least \$3 million be invested in each type of investment, that at least half the money be invested in Treasury notes and corporate bonds, and that the amount invested in corporate bonds not exceed twice the amount invested in Treasury notes. The annual yields for the various investments are 2% for Treasury notes, 4% for corporate bonds, and 5% for stocks. How should the money be allocated among the various investments to produce the greatest return?

Create a mathematical representation of this problem using just two variables. Clearly identify the objective function and the constraints. Graph the resulting feasible set.

You **do not** need to find the vertices of the feasible set, you **do not** need to find the maximum, and you **do not** need to solve the complete problem.

9. Consider the following problem:

The XYZ corporation plans to open three different types of fast-food restaurants. Type A restaurants require an initial cash outlay of \$600,000, need 15 employees, and are expected to make an annual profit of \$40,000. Type B restaurants require an initial cash outlay of \$400,000, need 9 employees, and are expected to make an annual profit of \$30,000. Type C restaurants require an initial cash outlay of \$300,000, need 5 employees, and are expected to make an annual profit of \$25,000. The XYZ corporation has \$48,000,000 available for initial outlays, does not want to hire more than 1000 employees, and would like to open at most 70 restaurants. How many restaurants of each type should be opened to maximize the expected annual profit?

Create a mathematical representation of this problem. Clearly identify the objective function and the constraints. Choose slack variables, and create a simplex tableau for this problem. Identify the first entry about which you would pivot the matrix.

You **do not** need to pivot the matrix. You **do not** need to find the maximum, and you **do not** need to solve the complete problem.

10. Consider the following problem:

A furniture manufacturer makes two types of furniture: chairs and sofas. For simplicity, divide the production process into three distinct operations- carpentry, upholstery, and finishing. Manufacture of a chair requires 6 hours of carpentry, 2 hours of upholstery, and 1 hour of finishing. Manufacture of a sofa requires 3 hours of carpentry, 6 hours of upholstery, and one hour of finishing. Due to limited availability of skilled labor as well as of tools and equipment, the factory has available each day 96 labor-hours for carpentry, 72 labor-hours for upholstery, and 18 labor-hours available for finishing. The profit per chair is \$80, and the profit per sofa is \$70. How many chairs and how many sofas should be produced each day to maximize the profit?

Completely solve this problem, using any method. Show all of your work.