

Final Exam
Math 111
May 17, 2005

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Graph the feasible set for the system of inequalities

$$\begin{cases} -3x + 4y \leq 8, \\ x + y \leq 9, \\ 3x + y \leq 21, \\ x \geq 0, y \geq 0. \end{cases}$$

Find the maximum of $f(x) = 3x + 11y$ on this feasible set.

2. Write the system of linear equations

$$\begin{cases} 9x + 7y = 6 \\ 5x + 4y = 11 \end{cases}$$

in the form $AX = B$ for matrices A , X , and B ; identify A , X , and B . Find A^{-1} and use it to solve the system.

3. Use the simplex method to maximize the function $10x + y$ subject to the constraints

$$\begin{cases} x + 2y \leq 10, \\ 3x + 4y \leq 6 \\ x, y \geq 0. \end{cases}$$

4. Use the simplex method to minimize the function $3x + 2y$ subject to the constraints

$$\begin{cases} x + y \geq 10, \\ x - y \leq 15, \\ x, y \geq 0. \end{cases}$$

5. Evaluate

a. $\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$,

b. $\binom{32}{2} + \binom{32}{3} + \binom{32}{4} + \binom{32}{5} + \dots + \binom{32}{32}$.

6. Suppose that an experiment consists of tossing a coin four times, and observing the sequence of heads and tails. [Order matters.]

- a. What is the sample space S ?
- b. Let E_1 be the event that two or more heads appear. What is E_1 ?
- c. Let E_2 be the event that two or more tails appear. What is E_2 ?
- d. What is $E_1 \cap E_2$? Give both the elements as well as a description in words.

§2 Comprehension

7. What is the geometric definition of slope?
8. What are the elementary row operations?
9. When are you allowed to multiply two matrices? Give an example of a pair of matrices A and B so that the product AB is defined (calculate it!) and so that the product BA is not defined.

10. Define the following terms:

- experiment
- outcome
- sample space
- event

§3 Application

11. For tax purposes, businesses must keep track of the current value of their assets. A common mathematical model is to assume that the current value y is related to the age x of the asset by a linear equation. A moving company buys a 40-foot van with a useful life of 5 years. After x months of use, the value y of the van is estimated by the linear equation

$$y = 25,000 - 400x.$$

- Sketch the graph of this linear equation.
- What is the value of the van after 5 years?
- What economic interpretation can be given to the y -intercept of the graph?

12. Consider the following problem:

A foreign-car dealer with warehouses in New York and Baltimore receives orders from dealers in Philadelphia and Trenton. The dealer in Philadelphia needs 4 cars and the dealer in Trenton needs 7. The New York warehouse has 6 cars and the Baltimore warehouse has 8. The cost of shipping a car from Baltimore to Philadelphia is \$120 per car, from Baltimore to Trenton is \$90 per car, from New York to Philadelphia is \$100 per car, and from New York to Trenton is \$70 per car. Find the number of cars to be shipped to each dealer from each warehouse to minimize shipping costs.

Create a mathematical representation of this problem using just two variables. Clearly identify the objective function and the constraints. Graph the resulting feasible set.

You **do not** need to find the vertices of the feasible set, you **do not** need to find the maximum, and you **do not** need to solve the complete problem.

13. Consider the following problem:

A baby products company makes car seats, strollers, and travel yards. A particular model of car seat requires 5 labor-hours for parts creation, 4 labor-hours of assembly, and 2 labor-hours of finishing and packaging. A particular model of stroller requires 3 labor-hours for parts creation, 3 labor-hours for assembly, and 1 labor-hour of finishing and packaging. A particular model of travel yard requires 2 labor-hours for parts creation, 3 labor-hours for assembly, and 1 labor-hour of finishing and packaging. There are 450 labor-hours available for parts creation, 380 for assembly, and 140 for finishing and packaging. If the profit per car seat is \$30, per stroller is \$10, and per travel yard is \$18, how many of each should be produced to maximize profit?

Create a mathematical representation of this problem. Clearly identify the objective function and the constraints. Choose slack variables, and create a simplex tableau for this problem. Identify the first entry about which you would pivot the matrix.

You **do not** need to pivot the matrix. You **do not** need to find the maximum, and you **do not** need to solve the complete problem.

14. Consider the following problem:

A furniture manufacturer makes two types of furniture: chairs and sofas. For simplicity, divide the production process into three distinct operations- carpentry, upholstery, and finishing. Manufacture of a chair requires 6 hours of carpentry, 2 hours of upholstery, and 1 hour of finishing. Manufacture of a sofa requires 3 hours of carpentry, 6 hours of upholstery, and one hour of finishing. Due to limited availability of skilled labor as well as of tools and equipment, the factory has available each day 96 labor-hours for carpentry, 72 labor-hours for upholstery, and 18 labor-hours available for finishing. The profit per chair is \$80, and the profit per sofa is \$70. How many chairs and how many sofas should be produced each day to maximize the profit?

Completely solve this problem, using any method. Show all of your work.

15. In how many different ways can the nine justices of the Supreme Court reach a 6-3 decision?

16. In how many ways can a committee of five senators be chosen from the 100 members of the U.S. Senate so that no two members are from the same state?
17. Two dice are rolled. What is the probability of their sum being precisely 7? What is the probability that their sum is either 2, 3, or 4?
18. A man, a woman, and their three children randomly stand in a row for a family picture. What is the probability that the parents will be standing next to each other?