

Midterm Exam #2

Math 263

October 15, 2001

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Answer the following:
 - a. Is 52 divisible by 13?
 - b. Does $7 \mid 34$?
 - c. Is 4 a factor of $6a \cdot 10b$?
2. Calculate:
 - a. $32 \operatorname{div} 9$.
 - b. $32 \operatorname{mod} 9$.

§2 Comprehension

3. What is the definition of a rational number? What is the definition of an irrational number? Be precise.
4. State precisely the unique factorization theorem.

§3 Application

5. Prove or disprove: If the sum of two integers is even, then so is the difference.
6. Prove or disprove: The sum of two rational numbers is rational.
7. Prove or disprove: For all integers a , b , and c , if $a \mid bc$, then either $a \mid b$ or $a \mid c$.
8. Prove that $\sqrt{2}$ is irrational.
9. Prove that $n^3 - n$ is divisible by 3 for every integer n .
10. Prove that $n^5 - n$ is divisible by 5 for every integer n .