

## Midterm Exam #3

Math 263

November 14, 2001

Name \_\_\_\_\_

*Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.*

### §1 Calculation

1. Simplify

a.  $\frac{8!}{7!}$

b.  $\frac{n!}{(n-3)!}$

c.  $\frac{((n+1)!)^2}{(n!)^2}$

2. Evaluate  $\sum_{m=0}^{127} \frac{3}{2^m}$ .

3. Let  $A$ ,  $B$ , and  $C$  be sets. For each of the following sets, draw a Venn diagram with the indicated set clearly labeled.

a.  $A \cap B$ ,

b.  $B^C$ ,

c.  $A - (B \cup C)$ .

### §2 Comprehension

4. State precisely the principle of mathematical induction. State precisely the principle of strong mathematical induction. Label each.

5. What is a partition? Give a precise definition, and include an example.

### §3 Application

6. Prove  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  for all positive integers  $n$ .

7. Prove that  $n^3 - n$  is divisible by 6 for each positive integer  $n$ . Include all the necessary details. ☺

8. Prove or disprove:  $1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$  for every positive integer  $n$ .

9. Prove or disprove:  $A \times (B \cap C) = (A \times B) \cap (A \times C)$ . for all sets  $A$ ,  $B$ , and  $C$ ,

10. Prove or disprove:  $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$  for all sets  $A$  and  $B$ .