

Midterm Exam #4

Math 263

December 7, 2001

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Let $A = \{2, 3, 5\}$ and $B = \{x, y\}$. Let p_1 and p_2 be the projections of $A \times B$ onto the first and second coordinates. That is, for each pair $(a, b) \in A \times B$, we have $p_1(a, b) = a$ and $p_2(a, b) = b$. Find the range of p_1 and the range of p_2 .
2. Let $a_k = 2a_{k-1} + k$ for all integers $k \geq 2$, and suppose that $a_1 = 1$. Find a_2 , a_3 , a_4 , and a_5 .
3. Let $b_k = 3b_{k-1} + 1$ for all $k \geq 1$, and suppose that $b_0 = 1$. Find a formula for all b_k .

§2 Comprehension

4. Define the following terms: injective, bijective, surjective.
5. What is the inverse of a function? Prove that every bijective function has an inverse function.
6. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$, and suppose that $g \circ f$ is onto. Is f onto? Prove or give a counterexample. Is g onto? Prove or give a counterexample.
7. What is a geometric sequence? In particular, what is the recursion relation, and what is the general term?

§3 Application

8. Let $\Sigma = \{0, 1\}$ and define $\ell : \Sigma^* \rightarrow \mathbf{Z}^{\text{nonneg}}$ by $\ell(s) =$ the length of s . Is ℓ one-to-one? Prove or give a counterexample. Is ℓ onto? Prove or give a counterexample.
9. Let $F : \mathcal{P}(\{a, b, c\}) \rightarrow \mathbf{Z}$ by $F(S) =$ the number of elements in S . Is F surjective? Prove or give a counterexample. Is F injective? Prove or give a counterexample.
10. A person saving for retirement makes an initial deposit of \$1000 to a bank account earning interest at a rate of 6% per year compounded monthly, at each month she adds an additional \$100 to the account. Use iteration to find an explicit formula for the amount of money A_n in the account after n months. How much will the account be worth after 40 years?