Midterm Exam #1
Math 263
September 20, 2002

Name_____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Prove or provide a counterexample: $(p \wedge \sim q) \vee p \equiv p$.

2. Prove or provide a counterexample: $\sim (p \to q) \equiv p \wedge \sim q$.

3. Write the contrapositive of the following statement, using "if-then" form. "Catching the 8:05 bus is sufficient for me to be on time for work."

§2 Comprehension

4. Consider the implication $p \to q$. What is its inverse? What is its converse? What is its contrapositive? Which are equivalent to the original implication?

5. Consider the argument
   You must pay a toll of $2.50 to cross the Bay Bridge
   John paid the $2.50 toll.
   $\therefore$ John crossed the Bay Bridge.
   a. What are the hypotheses?
   b. What is the conclusion?
   c. Is the argument valid or invalid?
   d. Assume the hypotheses are true. Is the conclusion true?

6. Define precisely the following terms.
   a. Statement
   b. Predicate
   c. Modus ponens
   d. Universal Modus tollens
   e. Disjunctive Addition

7. What are DeMorgan's Laws? Prove that they are true.

§3 Application

8. Use symbols to write the logical form of the following argument. If it is valid, give the rule that guarantees its validity; if it is invalid specify the fallacy.
   If Jules solved this problem correctly, then Jules obtained the answer 2.
   Jules obtained the answer 2.
   $\therefore$ Jules solved this problem correctly.

9. Use symbols to write the logical form of the following argument. If it is valid, give the rule that guarantees its validity; if it is invalid specify the fallacy.

   All honest people pay their taxes.
   John is not honest.
   $\therefore$ John does not pay his taxes.

10. What is the negation of the statement "For all $\varepsilon > 0$, there exists $\delta > 0$ so that if $0 < |x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$."?