Midterm Exam #3
Math 263
November 8, 2002

Name___________________________

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Evaluate
   a. $\sum_{k=1}^{5}(k-3)$
   b. $\prod_{i=1}^{4}\frac{i}{2}$

2. Evaluate
   a. $\sum_{k=1}^{n}\left(\frac{k}{k+1}-\frac{k+1}{k+2}\right)$
   b. $3+4+5+\cdots+100$.

3. Simplify
   a. $\frac{5!}{2!3!}$
   b. $\frac{n!}{(n-3)!3!}$.

§2 Comprehension

4. What is the principle of mathematical induction? What is the principle of strong mathematical induction? What is the well-ordering principle?

5. Analyze the following:
   Proposed Theorem: For all positive integers n, the number n^3-n+1 is divisible by 3.
   Proposed Proof: Let $P(n)$ be the predicate n, the number n^3-n+1 is divisible by 3. Now $P(n+1)$ is the statement that $(n+1)^3-(n+1)+1$ is divisible by 3. Since
   $$\begin{aligned}(n+1)^3-(n+1)+1 &= n^3+3n^2+3n+1-n-1+1\\ &= n^3-n+1+3\left(n^2+n\right)\end{aligned}$$
   we see that if $P(n)$ is true, then $P(n+1)$ is true. Thus, by induction, our theorem is proven.

§3 Application

6. Prove $1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$ for all positive integers *n*.

7. Prove $1+r+r^2+r^3+\cdots+r^n=\dfrac{r^{n+1}-1}{r-1}$ for all $r \neq 1$ and for all positive integers n.

8. Prove $2^{3n}-1$ is divisible by 7 for all positive integers n.

9. Prove $1+nx \leq (1+x)^n$ for all real numbers $x > -1$ and all integers $n \geq 2$.

10. Prove that every integer greater than 1 is divisible by a prime number.