

Midterm Exam #4

Math 263

December 4, 2002

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Let $\Sigma = \{a, b\}$, and define $g: \Sigma^* \rightarrow \mathbf{Z}$ as follows: for each string $s \in \Sigma$,

$$f(s) = \begin{cases} \text{the number of } b\text{'s to the left of the left-most } a \text{ in } s \\ 0 & \text{if } s \text{ contains no } a\text{'s} \end{cases}$$

Find $f(aba)$, $f(bbab)$ and $f(b)$. What is the range of f ?

2. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined by $f(x) = 2x - 9$. Prove that f is bijective, and find f^{-1} .

§2 Comprehension

3. What is the principle of mathematical induction? What is the principle of strong mathematical induction? What is the well-ordering principle?
4. What is a partition of a set? Give an example.
5. What is the definition of a function? What is the domain, the co-domain, and the range?

§3 Application

For 6-9 either prove the statement, or provide a counterexample.

6. For all sets A, B, C , we have $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
7. For any sets A and B , if $A \subset B$, then $\wp(A) \subset \wp(B)$.
8. Let $f: X \rightarrow Y$ be a function. For all sets $A, B \subset X$, if $A \subset B$, then $f(A) \subset f(B)$.
9. If $f: X \rightarrow Y$ and $g: Y \rightarrow X$ are functions, and suppose that $g \circ f = i_X$. Is $g = f^{-1}$?
10. Let A be a set, and define $f: A \times A \rightarrow A \times A$ by $f(x, y) = (y, x)$. Is f injective? Is f surjective? Is f bijective? If f is bijective, find f^{-1} .