

Exam #2
Math 273
October 25, 2005

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Differentiate each of the following.

- a. $y(x) = 4\pi^2 + x + 2x^2 - \sqrt{x}$
- b. $f(x) = e^x - \sin^{-1} x + \tan^{-1} x$
- c. $v(t) = \cos t + \tan t$.

2. Differentiate each of the following.

- a. $y(x) = \frac{x^2 + x - 2}{x^2 + 6}$
- b. $y(\theta) = \cot^2(\sin \theta)$
- c. $f(x) = x^{\sqrt{x}}$.

3. Find the equation of the tangent line to $y = \frac{\sqrt{x}}{x+1}$ at the point $(4, 2/5)$.

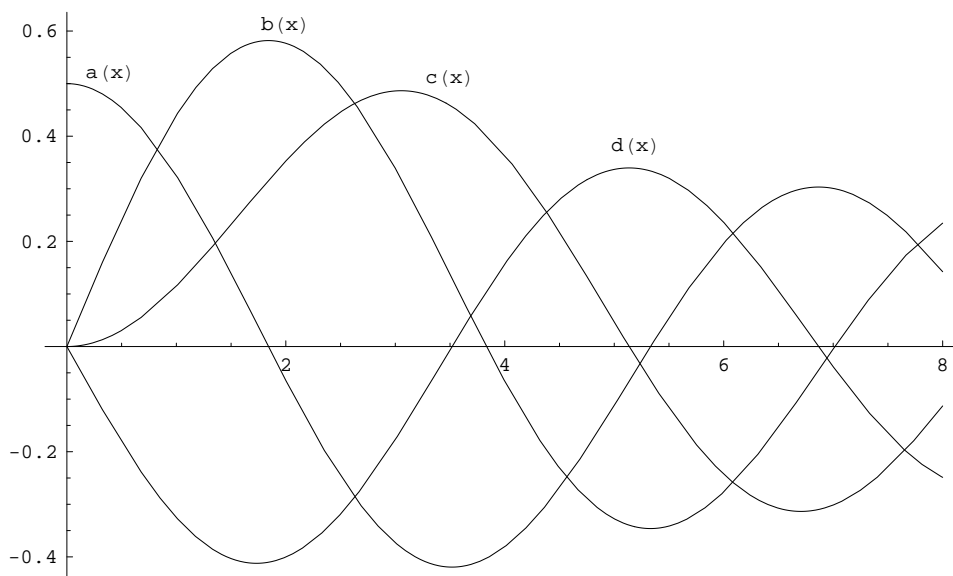
4. The graph of the curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ is called a lemniscate. Find dy/dx . Find the equation of the tangent line to the curve that passes through $(3, 1)$.

§2 Comprehension

6. Let $f(x)$ and $g(x)$ be differentiable. Prove that $(f + g)(x)$ is differentiable and that

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

7. Below are the graphs of a function $f(x)$, together with $f'(x)$, $f''(x)$ and an unrelated function $g(x)$. Identify each.



§3 Application

8. Boyle's Law states that when a sample of gas is compressed at constant temperature, the product of the pressure and the volume remain constant: $PV = C$. Find the rate of change of volume with respect to pressure. The isothermal compressibility is $\beta = -\frac{1}{V} \frac{dV}{dP}$. Show that $\beta = 1/P$.
9. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the radar station is increasing when it is 2 mi away from the station.
10. A runner sprints around a circular track of radius 100 m at a constant speed of 7 m/s. The runner's friend is standing at a distance of 200 m from the center of the track. How fast is the distance between the friends changing when the distance between them is 200 m.