

Exam #1
Math 273
September 28, 2006

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$.
2. Find all of the vertical and horizontal asymptotes of the curve

$$y = \frac{x^3 + 11}{x^3 - x}.$$

3. Find the equation of the tangent line to $y = \sqrt{x+1} - 2e^x$ at $(0, -1)$.
4. Find the derivative
 - (a) $y = 1/x^2$
 - (b) $y = \sqrt{x}(x+1)$
 - (c) $y = \frac{x^2 + x + 2}{x^3 - 6}$.

§2 Comprehension

5. What is the informal definition of limit? What is the precise definition of limit? Use the precise definition of limit to prove that $\lim_{x \rightarrow 3} x^2 = 9$.
6. Let

$$f(x) = \begin{cases} 2+x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$

Where is f continuous?

7. Prove that there is a number exactly one more than its cube.
8. What is the definition of the derivative? Give two different interpretations.

§3 Application

9. In the theory of relativity, the Lorentz contraction formula

$$L = L_0 \sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest, and c is the speed of light. Find $\lim_{v \rightarrow c^-} L$ and interpret the result. Why is a left hand limit necessary?

10. The velocity $v(t)$ of a falling raindrop at time t is given by

$$v(t) = u(1 - e^{-gt/u})$$

where g is the acceleration due to gravity, and u is called the *terminal velocity* of the raindrop. Evaluate $\lim_{t \rightarrow \infty} v(t)$. Explain the meaning of the term terminal velocity.