

Exam #2
Math 273
October 23, 2006

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Differentiate each of the following:

(a) $f(\theta) = \frac{\sec \theta}{1 + \tan \theta}$

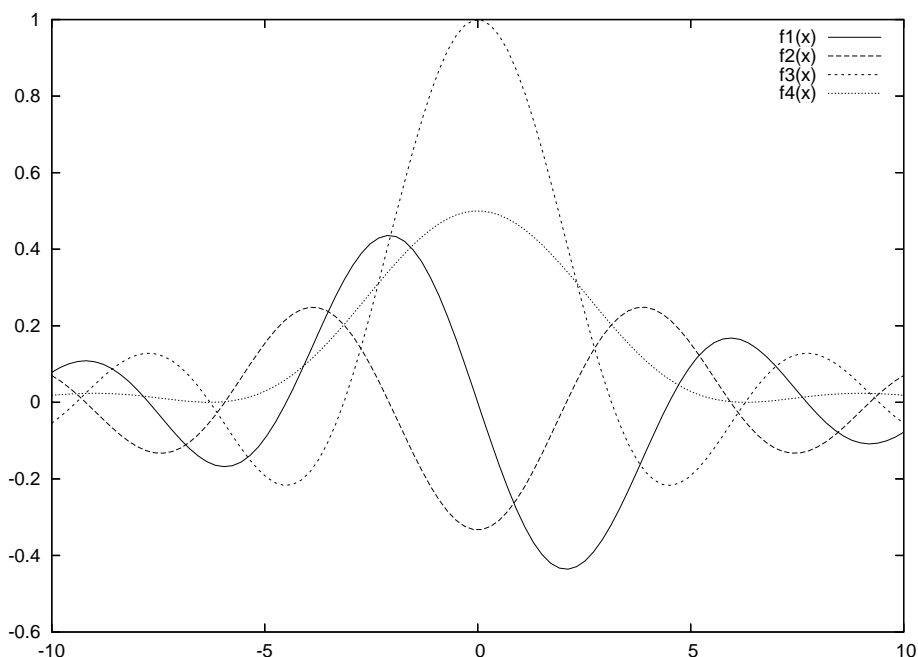
(b) $y(x) = \sqrt{e^x + 1}$

(c) $h(t) = \ln t + \frac{1}{\tan^{-1} t}$.

2. Find the (exact) equation of the tangent line to $y = \sin x$ at $x = \pi/3$. Draw a reasonably accurate graph of the tangent line and the function. What is the (approximate) value of the function $y = \sin x$ when $x = 1$? What is the exact and approximate value of the tangent line when $x = 1$.
3. Find the equation of the tangent line to $y^2 = x^3 + 3x^2$ at $(1, -2)$.
4. Find the derivative of $y(x) = x^{1/x}$.
5. Let $f(x) = \sin(2x)$. What is $f'(x)$? What is $f''(x)$? What is $f'''(x)$? What is $f^{(4)}(x)$? What is $f^{(42)}(x)$? Give a formula for $f^{(n)}(x)$ valid for all positive integers n .

§2 Comprehension

6. What is the derivative of $\sin^{-1} x$? Use implicit differentiation to prove your result.
7. Below are the graphs of four functions. One is the position of a particle, one is its velocity, one is its acceleration, and one function is unrelated. Identify each.



§3 Application

8. Boyle's Law states that, for a gas compressed at constant temperature, $PV = C$ where P is the pressure of the gas, V is the volume of the gas, and C is a constant. Find the rate of change of volume with respect to pressure. What is the significance of the sign of the result?
9. A man walks along a straight path at a speed of 3 ft/s. A searchlight is located on the ground 15 feet from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 30 feet from the searchlight?
10. A plane flying with a constant speed of 300 km/hr passes over Stephens Hall at an altitude of 1 km and climbs at an angle of 30° . At what rate is the distance from the plane to Stephens Hall changing one minute later?