

**Exam #3**  
Math 273  
November 16, 2006

Name \_\_\_\_\_

*All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.*

**§1 Calculation**

1. Find the exact absolute maximum and absolute minimum of  $f(x) = x^3 - 6x^2 + 9x + 2$  on the interval  $[-1, 4]$ . Justify your answer.
2. Find all of the exact local maxima and minima of  $p(x) = x\sqrt{5-x}$ . Determine where the function is increasing and decreasing. Justify your answer.
3. Let  $g(x) = x - 4\sin x$  be defined on the interval  $-\pi/2 < x < 3\pi/2$ . Find the intervals of concavity and the inflection points. Justify your answer.
4. Evaluate the limits analytically:

(a)  $\lim_{x \rightarrow (\pi/2)^+} \frac{\cos x}{1 - \sin x}$

(b)  $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$

(c)  $\lim_{x \rightarrow \infty} x^{1/x}$

5. Suppose that  $f''(\theta) = \sin \theta + \cos \theta$  and that  $f(0) = 3$ ,  $f'(0) = 4$ . Find  $f$ .

**§2 Comprehension**

6. What is the definition of absolute maximum? What is the definition of a local maximum? Give an example of a function with an absolute maximum but without a local maximum. Give an example of a function with a local maximum but without a global maximum.
7. State and prove Rolle's Theorem.

**§3 Application**

8. A right circular cylinder is inscribed in a sphere of radius  $R$ . Find the largest possible volume of such a cylinder.
9. A piece of wire 10m long is cut into two pieces. One is bent into a square and one is bent into a circle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) a minimum?
10. Suppose a particle moves in a straight line with constant acceleration  $a$  with initial velocity  $v_0$  and initial position  $s_0$ . Find an expression for the position  $s(t)$  for all later times  $t$ . Justify your answer.