

Final Exam
Math 273
December 13, 2006

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Find the limit:

(a) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^2}}{x}$.

(b) $\lim_{x \rightarrow 0} \frac{\sec x \tan x - x}{x^3}$.

(c) $\lim_{x \rightarrow 0^+} x^x$.

2. Find (exactly) all of the vertical and horizontal asymptotes of the curve $y = \frac{x^3 + 11}{x^3 - x}$. Explain why these are the asymptotes.

3. Differentiate:

(a) $y(x) = 2x\sqrt{x^2 + 1}$

(b) $h(t) = \ln t + \frac{1}{\tan^{-1} t}$

(c) $y = \frac{x^2 + x + 2}{x^3 - 6}$.

4. Find the equation of the tangent line to $y^3 + y^2 = x^2 + x$ at the point $(1, 1)$.

5. Evaluate:

(a) $\int (\sin x - e^x) dx$

(b) $\int_0^1 y(y^2 + 1) dy$

(c) $\int \frac{\sec \theta \tan \theta}{1 - \sec \theta} d\theta$.

6. Let $g(x) = x - 4 \sin x$ be defined on the interval $-3\pi/2 < x < 3\pi/2$. Find the intervals of concavity and the inflection points. Justify your answer.

§2 Comprehension

7. What is the informal definition of limit? What is the precise definition of limit? Use the precise definition of limit to prove that $\lim_{x \rightarrow 3} (4x - 3) = 9$.
8. Prove that there is a number exactly one more than its cube.
9. What is the definition of the derivative? Give two different interpretations.
10. What is the definition of absolute maximum? What is the definition of a local maximum? Give an example of a function with an absolute maximum but without a local maximum. Give an example of a function with a local maximum but without a global maximum.
11. What is the definite integral? What is the indefinite integral? What is the difference between them?

12. State both forms of the Fundamental Theorem of Calculus. Prove one.

§3 Application

13. In the theory of relativity, the Lorentz contraction formula

$$L = L_0 \sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest, and c is the speed of light. Find $\lim_{v \rightarrow c^-} L$ and interpret the result. Why is a left hand limit necessary?

14. A man walks along a straight path at a speed of 3 ft/s. A searchlight is located on the ground 15 feet from the path and is kept focuses on the man. At what rate is the searchlight rotating when the man is 30 feet from the searchlight?
15. A right circular cone is inscribed in a circle of radius R . Find the maximum volume of such a cone. What fraction of the total volume of the sphere is within this cone?
16. A car is traveling at 65 mi/hr when a driver sees an accident 150 ft ahead. How fast would the driver need to decelerate to avoid the accident? What fraction of the acceleration of gravity (32 ft/s²) would this be?