

## Midterm Exam #4

Math 273

May 9, 2001

Name \_\_\_\_\_

*Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.*

### §1 Calculation

1. Find the absolute maximum and absolute minimum for  $f(x) = \frac{x}{x^2 + 1}$  on  $[0, 2]$ .
2. Find the inflection points of  $f(x) = e^{1/x}$ .
3. Discuss  $f(x) = xe^x$ .
4. Evaluate  $\lim_{x \rightarrow 0^+} x \ln x$ .

### §2 Comprehension

5. What is Rolle's Theorem? Prove it.
6. What is Fermat's Theorem? Prove it.
7. Show that a cubic polynomial  $f(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$  has no local extrema if  $\beta^2 < 3\alpha\gamma$ .

### §3 Application

8. A farmer has 2000 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field with the largest area.
9. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) a minimum?
10. A right circular cylinder is inscribed in a sphere of radius  $r$ . Find the largest possible surface area of such a cylinder.