

## Final Examination

Math 273

May 17, 2001.

Name \_\_\_\_\_

*Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.*

### §1 Calculation

1. Evaluate the following limits, or explain why it does not exist.

a.  $\lim_{x \rightarrow -3} \frac{x^2 - x + 12}{x + 3}$

b.  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 4} - 2}{x}$ .

c.  $\lim_{x \rightarrow \infty} x^2 \sin\left(\frac{1}{x}\right)$ .

d.  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ .

2. Differentiate. Simplify your answer.

a.  $f(x) = x^3 - \frac{1}{\sqrt{x}} + \ln x + \cos x - e^3$ .

b.  $g(\theta) = \sin(2\theta) + \left(\frac{1 - \cos 2\theta}{2}\right)^2$ .

c.  $h(y) = 4\pi^2 + y \ln y + \sqrt{y}e^y$ .

d.  $f(x) = \frac{x}{x + \frac{c}{x}}$ .

3. Find the equation of the tangent line to the astroid  $x^{2/3} + y^{2/3} = 4$  that passes through the point  $(-3\sqrt{3}, 1)$ .

4. Discuss  $f(x) = xe^x$ .

5. Evaluate.

a.  $\frac{d}{dx} \int_0^x \frac{1}{1+t^3} dt$ .

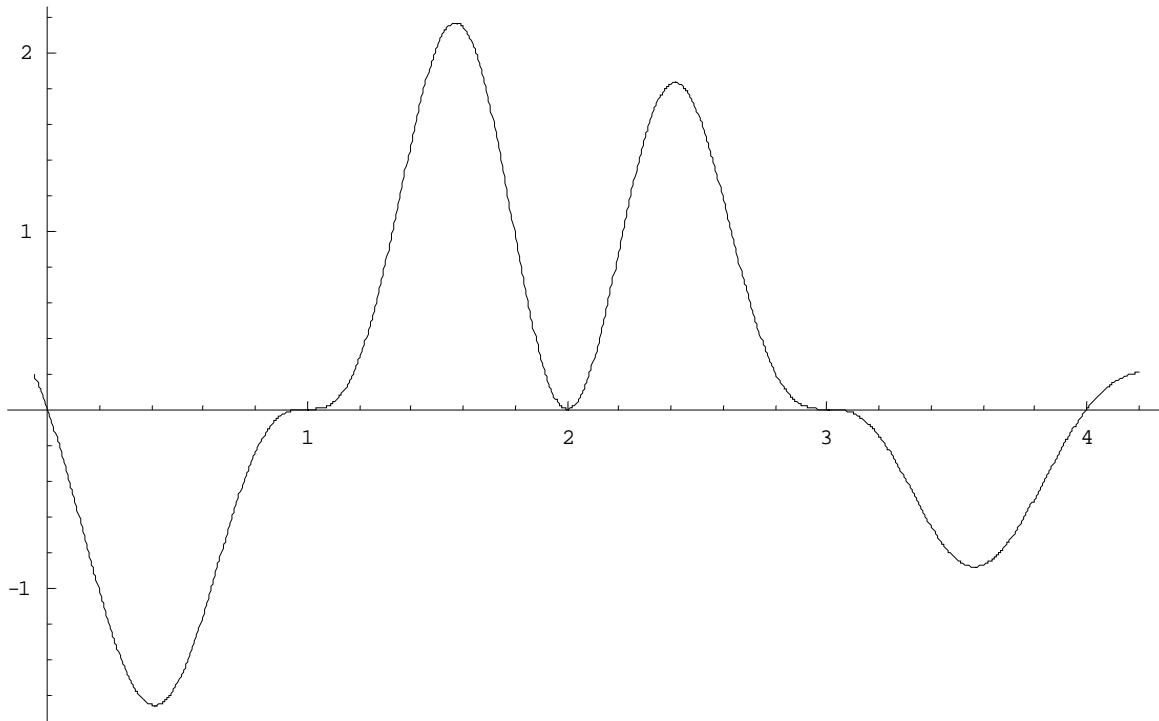
b.  $\int \cos 2\theta d\theta$

c.  $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ .

d.  $\int \frac{x}{1+x^4} dx$ .

## §2 Comprehension

6. The graph of the derivative  $f'(x)$  appears below. (*The graph of  $f(x)$  does not appear!*). Answer the following. Give exact answers where possible, but approximate answers are acceptable.
- Where does  $f(x)$  have critical points?
  - Where does  $f(x)$  have local maxima? Where does  $f(x)$  have local minima?
  - Where does  $f(x)$  have inflection points?
  - Where is  $f(x)$  concave up? Where is  $f(x)$  concave down?
  - Sketch the graphs of two functions whose derivative is  $f'(x)$ .



- Give both an intuitive definition and a rigorous definition of a limit
- Give a rigorous definition of the derivative. Give two interpretations of the significance of the derivative.
- Sketch the graph of a function whose first derivative and second derivative are always negative.
- Show that the equation  $x^5 - 6x + c = 0$  has at most one root in the interval  $[-1, 1]$ .
- What is the definition of  $\int f(x) dx$ ? What is the definition of  $\int_a^b f(x) dx$ ? Be precise.
- What is the first version of the Fundamental Theorem of Calculus? What is the second version? Explain why the first version is true.

### §3 Application

13. A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 feet from the path and kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight? Give an exact answer, and include your units.
14. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of  $2 \text{ m}^3$ , find the rate at which the water level is rising when the water is 3 m deep. Give an exact answer, and include your units.
15. A farmer has 2000 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the exact dimensions of the field with the largest area?
16. A right circular cylinder is inscribed in a cone with height  $h$  and base radius  $r$ . Find the largest possible volume of such a cylinder.
17. An object is being accelerated at a rate  $a(t) = 9 \sin t$ , with initial velocity  $v_0 = 1$  and initial position  $x_0 = 0$ . Find the position of the object for all times  $t$ .