

**Midterm Examination #2**  
Math 273 Calculus 1  
Wednesday, March 14, 2001

Name \_\_\_\_\_

The use of graphing calculators is permitted.

**§1 Computation:**

1) Evaluate the following limits.

a)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x}$ .

b)  $\lim_{x \rightarrow -3} \frac{x^2 + 5x + 6}{x + 3}$ .

c)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$ .

2) Find the vertical and horizontal asymptotes, if any, of the following functions. Check your results by including an appropriate graph.

a)  $f(x) = \frac{\sqrt{x^2 + 1}}{3x - 5}$ .

b)  $f(x) = \frac{x}{x + 4}$ .

3) If  $G(x) = \frac{x}{1 + 2x}$ , use the definition of derivative to find  $G'(a)$ , and use it to find an equation

of the tangent line to the curve  $y = \frac{x}{1 + 2x}$  at the point  $\left(-\frac{1}{4}, -\frac{1}{2}\right)$ . (Failure to use the definition of derivative will result in no points being awarded!)

4) Use the squeeze theorem to evaluate  $\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right)$ .

**§2 Comprehension:**

5) Give both an intuitive definition and a rigorous definition of a limit

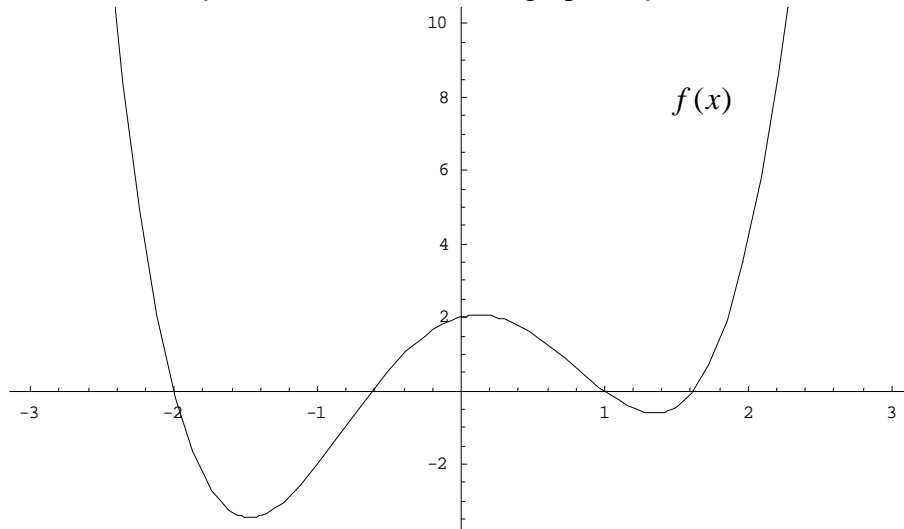
6) Give a rigorous definition of the derivative. Give two interpretations of the significance of the derivative.

7) For each of the following two statements, either provide a proof showing the statement is true, or a counterexample showing the statement is false.

a) If  $f$  is differentiable at  $a$ , then  $f$  is continuous at  $a$ .

b) If  $f$  is continuous at  $a$ , then  $f$  is differentiable at  $a$ .

8) The graph of a function  $f(x)$  is shown. Sketch a graph of  $f'(x)$ .



### §3 Applications:

9) The gravitational force exerted by Earth on a unit mass at a distance  $r$  from the center of the planet is

$$F(r) = \begin{cases} \frac{GMr}{R^3} & \text{if } r < R, \\ \frac{GM}{r^2} & \text{if } r \geq R. \end{cases}$$

where  $M$  is the mass of the earth,  $R$  is its radius, and  $G$  is the gravitational constant. Is  $F$  a continuous function of  $r$ ? Explain your answer.

10) A tank contains 5000 L of pure water. Brine that contains 25 g of salt per liter of water is pumped into the tank at a rate of 20 L/min. Find the concentration of salt after  $t$  minutes (in grams per liter). What happens to the concentration as  $t \rightarrow \infty$ ?