

Midterm Exam #2

Math 273-004

March 13, 2002

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

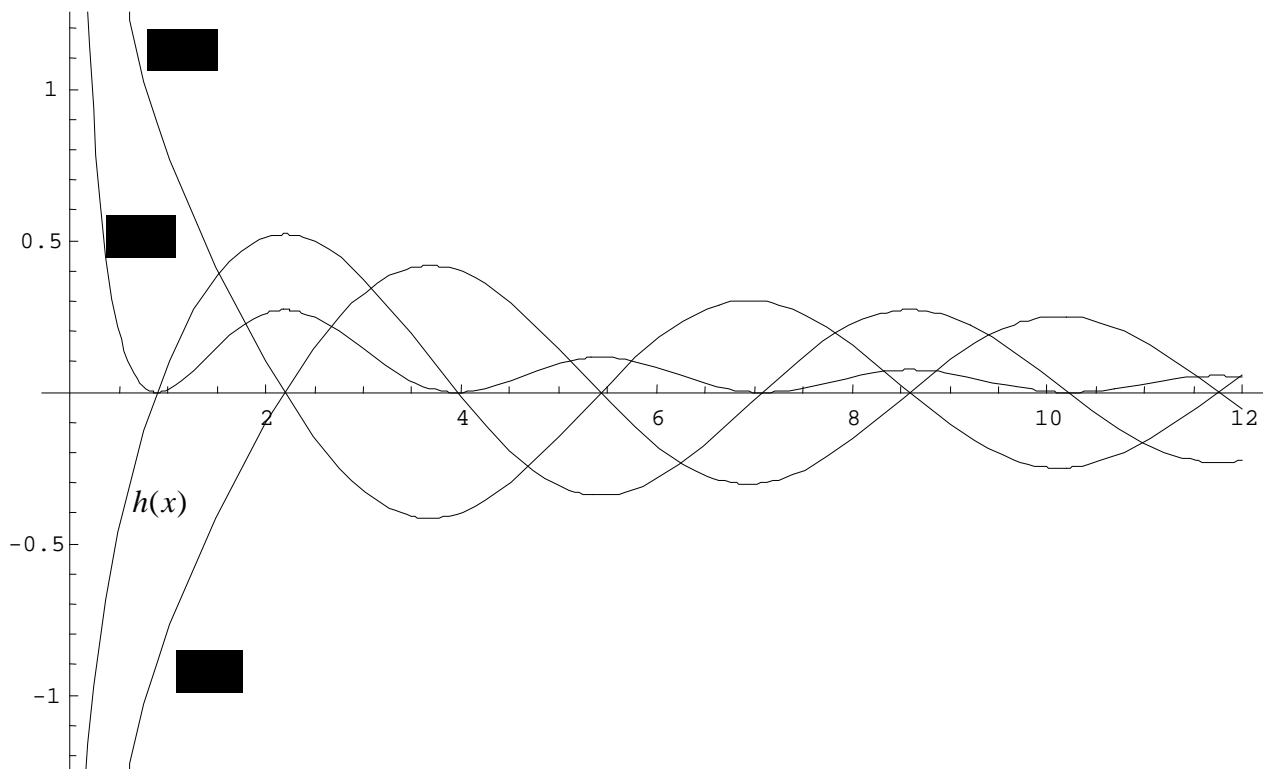
§1 Calculation

1. Evaluate exactly $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 16} - 4}{t^2}$. [Note: Using a graph or table does not qualify as “Evaluate exactly.”]
2. Evaluate exactly $\lim_{x \rightarrow 0^+} x^2 \sin\left(\frac{1}{x + \sqrt{x}}\right)$. [Note: Using a graph or table does not qualify as “Evaluate exactly.”]
3. Let $f(x) = \frac{\sqrt{3x^2 + 2}}{2x - 1}$. Find (exactly) all of the vertical and horizontal asymptotes of $f(x)$. [Note: Using a graph or table does not qualify as “exactly”.]
4. Calculate the derivative. [Use any method.]
 - a. $f(x) = e^x + x^2 + \sqrt{x} - \frac{1}{x}$.
 - b. $f(x) = xe^x$.
 - c. $f(x) = \frac{x}{x^2 + 1}$.

§2 Comprehension

5. What is the precise definition of $\lim_{x \rightarrow a} f(x) = L$? Use it to prove that $\lim_{x \rightarrow 2} (3x - 1) = 5$.
6. Give a precise definition of what it means for a function to be continuous at a point. Prove that if a function is differentiable at a point, then it is continuous at that point.
7. Give a precise definition of derivative. Give two different interpretations of its significance. Use the definition of derivative to find the derivative of $f(x) = x^2$ at $x = a$.

8. Four functions $f(x)$, $g(x)$, $h(x)$, and $k(x)$ are shown in the graph below. One of these functions is the derivative of one of the others. Identify which. Explain your reasoning.



§3 Application

9. The position of a particle after t seconds is given by the position function $s = f(t) = t^2 - 4t$. Find the velocity as a function of time. What is the velocity when $t = 1$? What does the sign of the velocity indicate?
10. Find the equation(s) of the tangent line (or lines) to the curve $y = \frac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.