Midterm Exam #3
Math 273-004
April 10, 2002

Name_____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Differentiate $f(x) = \dfrac{\sec x}{1 + \tan x}$. Where does $f(x)$ have a horizontal tangent?

2. Differentiate:
   a. $f(x) = \cos(\sin x)$
   b. $g(x) = \ln(\sec x + \tan x)$
   c. $h(x) = \tan^{-1} x$

3. Find the equation of the tangent line to $2\left(x^2 + y^2\right)^2 = 25\left(x^2 - y^2\right)$ at $(3,1)$.

4. Differentiate $y = (\cos x)^x$.

5. Find the (exact!) absolute maximum and absolute minimum of $f(x) = e^{-x} - e^{-2x}$ on the interval $[0,1]$.

§2 Comprehension

6. State precisely the Chain Rule. Use the Chain Rule to prove that the derivative of an even function is an odd function.

7. State and prove Fermat's Theorem.

§3 Application

8. The frequency of vibrations in a vibrating violin string is given by $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}}$ where L is the linear density of the string, T is its tension, and ρ is its linear density.
   a. Find the rate of change of the frequency with respect to
      i. the length (when T and ρ are constant),
      ii. the tension (when L and ρ are constant),
      iii. the linear density (when L and T are constant).
   b. The pitch of a note is determined by the frequency f; the higher the frequency, the higher the pitch. Use the signs of the derivatives in part a to determine what happens to the pitch of a note
      i. when the effective length of a string is decreased by placing a finger on the string so that a shorter portion of the string vibrates, [*problem continues*]

ii. when the tension is increased by turning a tuning peg,
iii. when the linear density is increased by changing to another string.

9. The minute hand on a clock is 8mm long, and the hour hand is 4mm long. How fast (exactly!) is the distance between the tips of the hands changing at 1:00?

10. When air expands aidiabatically, its pressure P and volume V are related by $PV^{1.4} = C$ for some constant C. Suppose that at a certain instant the volume is 400 cm^3 and the pressure is 80 kPa and is decreasing at a rate of 10 kPa/min. At what rate is the volume increasing at this instant?