

Midterm Exam #4

Math 273

May 1, 2002

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

- Find the (exact!) maximum and minimum values of $f(x) = \sin x + \cos x$ on $[0, \pi/3]$.
- Let $f(x) = \frac{1-x^2}{1+x^2}$. Find the intervals of concavity and the inflection points [exactly!].
- Evaluate the following limits exactly
 - $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$
 - $\lim_{x \rightarrow \pi^-} \frac{\sin x}{1 - \cos x}$
 - $\lim_{x \rightarrow \infty} \left(x - \sqrt{x^2 - 1} \right)$
 - $\lim_{x \rightarrow \infty} x^{1/x}$
- Discuss $f(x) = xe^{-x^2}$. In particular,
 - find the exact domain,
 - find the exact intercepts,
 - determine the symmetry,
 - find the exact asymptotes,
 - find the exact intervals of increase/decrease,
 - find the exact local extrema,
 - determine exactly the concavity and points of inflection, and
 - sketch the curve.

§2 Comprehension

- State and prove the Mean Value Theorem.
- Show that the equation $x^5 - 6x + c = 0$ has at most one real solution in $[-1, 1]$, regardless of the value of c .
- Suppose that the cubic polynomial $f(x) = Ax^3 + Bx^2 + Cx + D$ has extrema at the two distinct points x_1 and x_2 . Show that $f(x)$ has an inflection point midway between x_1 and x_2 .

§3 Application

8. At noon, ship A is 150 km west of ship B. Ship A is sailing west at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 p.m.?
9. A box with a square base and open top has a volume of $32,000 \text{ cm}^3$. Find the dimensions of the box that minimize the amount of material used.
10. A right circular cone is inscribed in a sphere of radius R . Find the largest volume of such a cone. What fraction of the volume of the sphere is occupied by the cone?