

Final Examination

Math 273

May 15, 2002

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Find the limits (Exactly!):

a. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

b. $\lim_{t \rightarrow 0} \left[\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right]$

c. $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \theta}{\theta^3}$

d. $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$.

2. Find the derivatives:

a. $f(x) = \pi^2 + x^2 - \ln x + \sec x - e^x$

b. $f(x) = \frac{1 + 2x \sin x}{1 + x^2}$

c. $f(x) = \sqrt[3]{1 + \tan x}$

d. $f(x) = x^x$.

3. Find the integrals:

a. $\int_0^1 (1 - 2x - 3x^2) dx$

b. $\int x(1 + 2x^4) dx$

c. $\int \frac{1+x}{1+x^2} dx$

d. $\int_0^4 \frac{x}{\sqrt{1+2x}} dx$

4. Discuss $f(x) = xe^{-x^2}$. In particular,

- find the exact domain,
- find the exact intercepts,
- determine the symmetry,
- find the exact asymptotes,
- find the exact intervals of increase/decrease,
- find the exact local extrema,
- determine exactly the concavity and points of inflection, and
- sketch the curve.

5. Find the equation of the tangent line to $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at $(3,1)$.

§2 Comprehension

6. Give an intuitive definition of limit. Give the precise definition of $\lim_{x \rightarrow a} f(x) = L$? Use it to prove that

$$\lim_{x \rightarrow 2} (3x - 1) = 5.$$

7. Give a precise definition of derivative. Give two different interpretations of its significance. Use the definition of derivative to find the derivative of $f(x) = x^2$ at $x = a$.

8. What is the definite integral? What is the indefinite integral? Use the definition(s) to evaluate $\int_0^2 x^2 dx$.

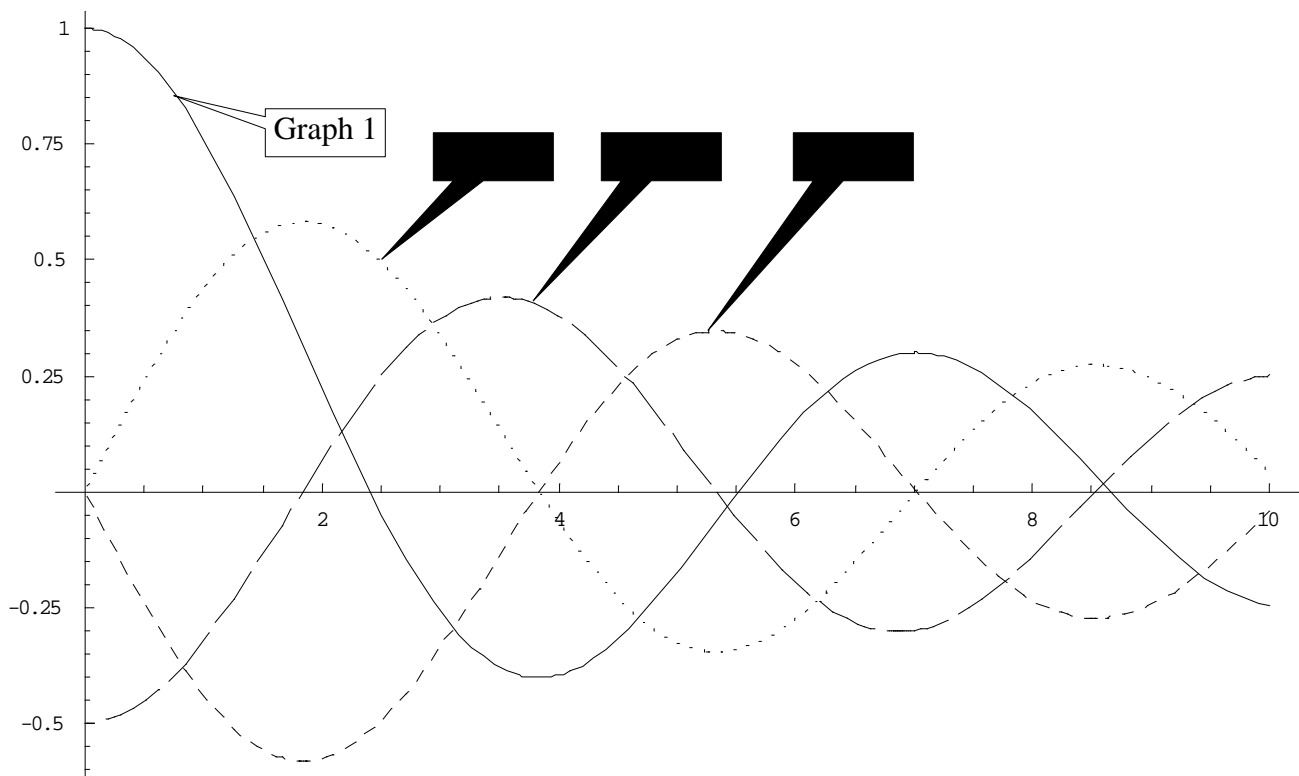
Do not use the fundamental theorem of calculus! You may use the fact that

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

9. State the Mean Value Theorem. Prove that if $g(x)$ and $h(x)$ are antiderivatives of a function $f(x)$ on the interval (a,b) , then $g(x) - h(x)$ is constant on (a,b) .

10. State and prove both versions of the fundamental theorem of calculus.

11. Below are the graphs of a function $f(x)$, its derivatives $f'(x)$ and $f''(x)$, together with another function $g(x)$. Identify which is the graph of $f(x)$, which is $f'(x)$, which is $f''(x)$, and which is $g(x)$.



§3 Application

12. At noon, ship A is 150 km west of ship B. Ship A is sailing west at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 p.m.?
13. A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path, and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft away from the point on the path closest to the searchlight? Include the units of your answer.
14. Suppose 1200 cm^2 of material are available to make a box with square base and open top. What is the largest possible volume of the box?
15. A right circular cone is inscribed in a sphere of radius R . Find the largest volume of such a cone. What fraction of the volume of the sphere is occupied by the cone?
16. An object is thrown upward with a velocity of 48 ft/s from the top of a tower that is 112 feet above the ground. When does it hit the ground? How fast is it going at impact?