Midterm Exam #1
Math 273
March 6, 2003

Name_____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Evaluate the following limits exactly:
   a. $\lim_{x\to -3} \dfrac{x^2 - x + 12}{x+3}$,
   b. $\lim_{h\to 0} \dfrac{(2+h)^3 - 8}{h}$,
   c. $\lim_{x\to 2} \dfrac{\left(\dfrac{1}{x} - \dfrac{1}{2}\right)}{x-2}$.

2. Find all of the horizontal and vertical asymptotes of $f(x) = \dfrac{x^3+1}{x^3+x}$. Justify your answer.

3. Evaluate the following limits exactly
   a. $\lim_{x\to\infty} \dfrac{x+4}{x^2 - 2x + 5}$,
   b. $\lim_{x\to\infty} \dfrac{x^7 - 1}{x^6 + 1}$,
   c. $\lim_{x\to\infty} \left(\sqrt{9x^2 + x} - 3x\right)$.

§2 Comprehension

4. What is the precise definition of function? Give an example of a function whose range and co-domain are different. In your example, identify the domain, co-domain and range.

5. What is the horizontal line test? What is the domain and range of the function $\sin^{-1} x$? Explain your answer.

6. Give an informal definition of limit. Give the precise definition of limit. Use the precise definition of limit to prove that $\lim_{x\to 3}(2x+1) = 7$.

7. Define precisely what it means for a function to be continuous at a point. At what points is the function $f(x) = x^2 + \sqrt{x}$ continuous? Justify your answer.

§3 Application

8. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

9. At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.
    a. Express the water pressure as a function of the depth below the ocean surface.
    b. At what depth is the pressure 100 lb/in^2?

10. An isotope of radon ^{222}Rn has a half-life of 3.8 days. A sample of radon contains 2g.
    a. Find the amount remaining at the end of one week.
    b. How long will it take until only 0.01g of radon remains?