

Midterm Exam #3

Math 273

May 1, 2003

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

- Find the exact points where $f(x) = x^3 + \frac{1}{2}x^2 - x + 1$ attains its local maxima and minima on the interval $[-1, 1]$.
- Find the absolute extrema and the local extrema of $f(x) = x + 2\sin x$ on the interval $[0, 2\pi]$. Label each.
- Let $f(x) = \frac{1+x^2}{1-x^2}$. Find (exactly!)
 - The horizontal asymptotes
 - The vertical asymptotes
 - The local extrema
 - The intervals of increase and decrease
 - The intervals of concavity
 - The inflection points
- Evaluate
 - $\lim_{x \rightarrow 0} \frac{\tan \alpha x}{x}$
 - $\lim_{x \rightarrow \infty} x e^{-x}$
 - $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x + 1} - \sqrt{x^2 + x})$
 - $\lim_{x \rightarrow 0^+} x^{\ln x}$

§2 Comprehension

- State and prove Fermat's Theorem.
- If $f(1) = 10$ and $f'(x) \geq 2$ on the interval $[1, 4]$, what is the smallest value that $f(4)$ can have? Prove your result.
- What is the first derivative test? What is the second derivative test?

§3 Application

8. Find the point on the line $2x + y = 5$ closest to the origin.
9. Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius r .
10. A right circular cylinder is inscribed in a cone with height h and base radius r . Find the maximum volume of the cylinder.