

**Final Exam**  
Math 273  
May 14, 2003

Name \_\_\_\_\_

*Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.*

**§1 Calculation**

1. Evaluate the following limits exactly:

- a.  $\lim_{x \rightarrow 2} \frac{\left(\frac{1}{x} - \frac{1}{2}\right)}{x - 2}$
- b.  $\lim_{x \rightarrow \infty} \frac{x^2 - 7x + 6}{\sqrt{x^4 - 5x^2 + 11}}$
- c.  $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$ .
- d.  $\lim_{x \rightarrow 0} \frac{\tan \alpha x}{x}$

2. Differentiate

- a.  $f(x) = e^2 + e^x + \sin x + \ln x + \sin^{-1} x$
- b.  $f(x) = \frac{2x \cos x}{\sqrt{x+1}}$
- c.  $f(x) = \tan^2(\cos 2x)$
- d.  $f(x) = \int_0^x (2t \sin t)^2 dt$

3. Find the integrals:

- a.  $\int_0^1 (6 - x + x^2) dx$
- b.  $\int \left( x^3 - \sin x + \frac{1}{x} + \sec^2 x + \frac{1}{1+x^2} \right) dx$
- c.  $\int \frac{1+x}{1+x^2} dx$
- d.  $\int 7x(1+x^4) dx$

4. Find the equation of the tangent line to  $x^3 + y^3 = 2xy$  at  $(1,1)$ .

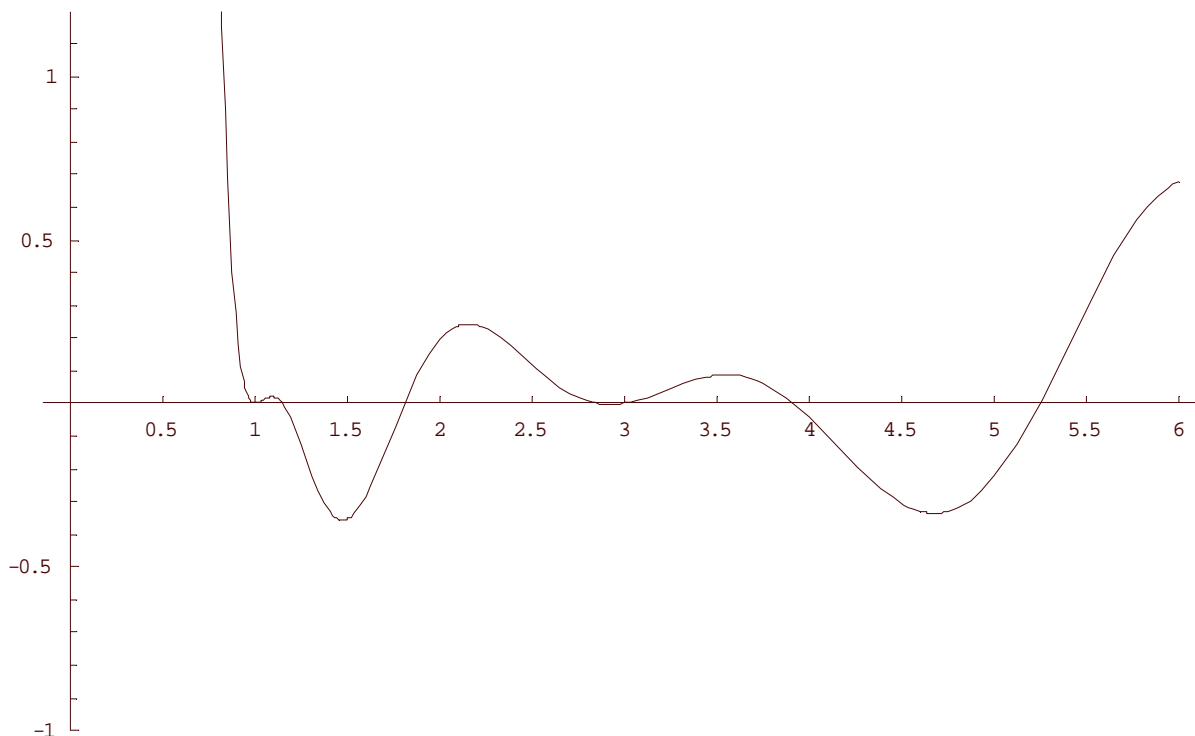
5. Let  $f(x) = xe^{2x}$ . Find (exactly!)

- a. The domain
- b. The horizontal asymptotes
- c. The vertical asymptotes
- d. Symmetries (if any)

- e. The local extrema
- f. The intervals of increase and decrease
- g. The intervals of concavity
- h. The inflection points

## §2 Comprehension

6. Give an informal definition of limit. Give the precise definition of limit. Use the precise definition of limit to prove that  $\lim_{x \rightarrow 3} (2x - 1) = 5$ .
7. What is the precise definition of the derivative of a function? Use the precise definition to find the derivative of  $f(x) = \sqrt{x}$  at  $x = a$ .
8. What is the definite integral? What is the indefinite integral? Use the definition(s) to evaluate  $\int_0^3 x^3 dx$ .  
Do not use the fundamental theorem of calculus! You may use the fact that  $1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$ .
9. Below is a graph of the derivative  $f'(x)$  of a function.
- a. What are the (approximate) critical points of  $f(x)$ ?
  - b. What are the (approximate) local maxima and minima of  $f(x)$ ?
  - c. At the point  $x = 5$ , is the function  $f(x)$  increasing or decreasing?
  - d. At the point  $x = 2$ , is the function  $f(x)$  concave up, or concave down?



The graph of  $f'(x)$ . Note that the graph of  $f(x)$  is **NOT** given

10. If  $f(1) = 10$  and  $f'(x) \geq 2$  on the interval  $[1, 4]$ , what is the smallest value that  $f(4)$  can have? Prove your result.
11. State both versions of the fundamental theorem of calculus. Prove one. (Your choice).

### §3 Application

12. At the surface of the ocean, the water pressure is the same as the air pressure above the water,  $15 \text{ lb/in}^2$ . Below the surface, the water pressure increases by  $4.34 \text{ lb/in}^2$  for every 10 ft of descent.
- Express the water pressure as a function of the depth below the ocean surface.
  - At what depth is the pressure  $100 \text{ lb/in}^2$ ?
13. The position of a particle is given by the equation  $s(t) = t^3 - 6t^2 + 9t$ , where the time  $t$  is measured in seconds, and the position  $s$  is measured in meters.
- What is the velocity at time  $t$ ?
  - When is the particle at rest?
  - When is the particle moving forward?
  - What is the total distance the particle travels during the first five seconds?
14. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs towards first base with a speed of 24 ft/s. How fast is the batter's distance from second base changing when he is two-thirds of the way to first base?
15. A runner runs around a circular track of radius 300 ft at a constant speed of 20 ft/s. The runner's friend is 500 ft from the center of the track. How fast is the distance between the friends changing when they are 400 feet apart?
16. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius  $r$ .
17. A right circular cylinder is inscribed in a cone with height  $h$  and base radius  $r$ . Find the maximum volume of the cylinder.