

Midterm Examination #2
Math 273 Calculus 1
Thursday, April 8, 1999

Name _____

The use of graphing calculators is permitted.

§1 Computation:

1) Differentiate the following functions.

- a) $f(x) = \frac{1}{x}$.
- b) $g(t) = t^2 - 2t + 1$.
- c) $h(t) = \pi \cos t$.
- d) $\phi(s) = 3 \sin s - \frac{3}{4} e^s$.

2) Differentiate the following functions.

- a) $f(x) = \frac{\cos x}{e^x}$.
- b) $y(x) = \frac{x^3 + 3x + 2}{x^2 - 1}$.
- c) $g(t) = \sqrt{t} + \ln t + 4 \sec t$.

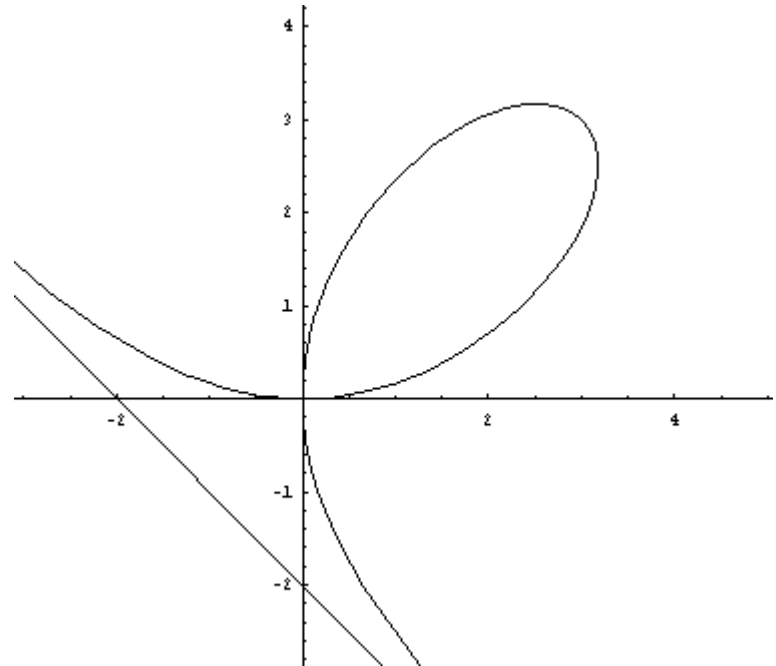
3) Differentiate the following functions.

- a) $g(t) = (e^t + e^{-t})^3$.
- b) $f(x) = \ln \left(\frac{x^2}{\sqrt{x^2 + 1}} \right)$.
- c) $y(\theta) = \frac{1}{4} \sin^2 2\theta$.

4) The Foilum of Descartes has equation

$$x^3 + y^3 - 6xy = 0$$

and whose graph is given on the right. Find the tangent line to this graph that passes through the point $(\frac{4}{3}, \frac{8}{3})$

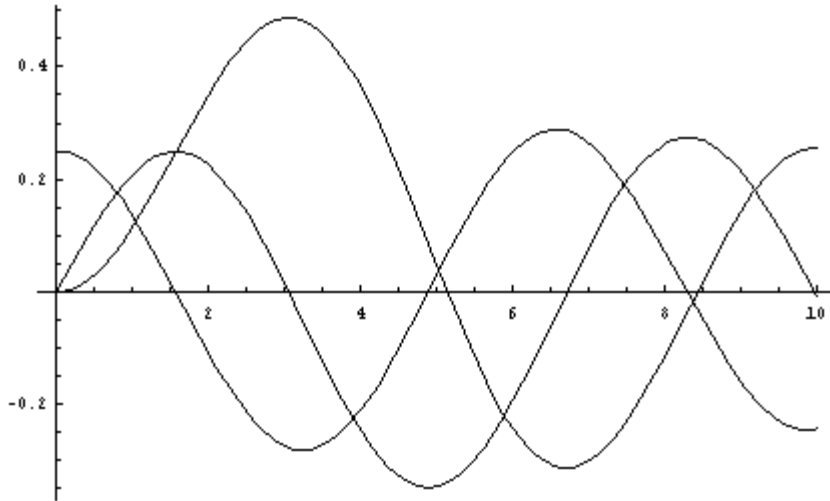


§2 Comprehension:

5) What is the relationship between the tangent line of a function and the derivative of a function. Explain your answer, and include a graph. Also, include an example.

6) Give a rigorous definition of the derivative. Use the rigorous definition to calculate the derivative of $f(x) = x^3 + 2x$.

- 7) If a function is continuous at a point, must it also be differentiable at that point? Prove your result, or give a counterexample. If a function is differentiable at a point, must it be continuous at that point? Prove your result, or provide a counterexample.
- 8) Here is the graph of a function, its first derivative, and its second derivative. Identify each.



§3 Applications:

- 9) A spherical balloon is inflated with gas at a rate of 20 cubic feet per minute. How fast is the radius of the balloon changing when the radius is 1 foot? When the radius is 2 feet?
- 10) A conical tank, with vertex down, is 10 feet across at the top, and 12 feet deep. If water is flowing into the tank at the rate of 10 cubic feet per minute, find the rate of change of the height of the water at the moment when the water is 8 feet deep.