

Final Examination
Math 273 Calculus 1
Tuesday, May 18, 1999

Name _____

The use of graphing calculators is permitted.

§1 Computation:

1) Evaluate the following limits.

a) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$.

b) $\lim_{x \rightarrow \infty} \frac{x^2 - x}{\sqrt{2x^4 + x}}$.

c) $\lim_{x \rightarrow 0^+} \frac{x+1}{\sqrt{x}}$.

d) $\lim_{x \rightarrow 2} \frac{1}{(x-1)^2}$.

2) Differentiate the following functions.

a) $f(x) = x^3 + e^x - \ln x + \cos x$

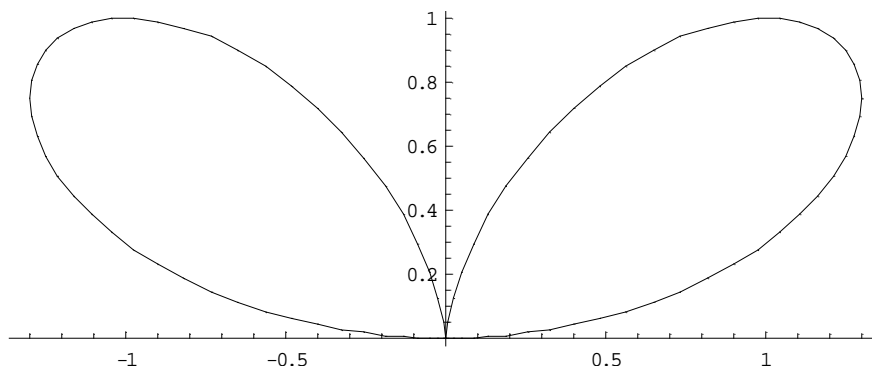
b) $f(x) = \left(x^2 + \frac{1}{x}\right)^5$

c) $f(x) = x^2 \ln(1-x)$.

d) $y(\theta) = \frac{1 + \sin \theta}{1 - \sin \theta}$.

3) The Bifolium, graphed below, has equation $(x^2 + y^2)^2 = 4x^2y$ and whose graph given on the right. Find the tangent line to this graph that passes through the point

$$\left(\frac{1+\sqrt{2}}{2}, \frac{1}{2}\right).$$



4) Find the extrema of $f(x) = (x^2 - 4)^{2/3}$ on the interval $[-3, 3]$. Indicate where the function is increasing, and where it is decreasing.

5) Analyze the graph of $f(x) = \frac{x^2 - 2x + 4}{x - 2}$. In particular, find its intercepts, asymptotes, local extrema, and points of inflection.

6) Evaluate.

a) $\int x(x^2 + 1)^2 dx$.

b) $\int \sqrt{2x - 1} dx$.

c) $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right) dx$.

d) $\int_1^9 \frac{1}{\sqrt{x}(1 + \sqrt{x})^2} dx$.

§2 Comprehension:

7) Give both an intuitive definition and a rigorous definition of a limit. Use the rigorous definition to prove that $\lim_{x \rightarrow 2} (2x - 1) = 3$.

8) Give a rigorous definition of the derivative. Use the rigorous definition to calculate the derivative of $f(x) = x^3 + 2x$.

9) What is the significance of the derivative? Give at least three different applications of the idea of the derivative.

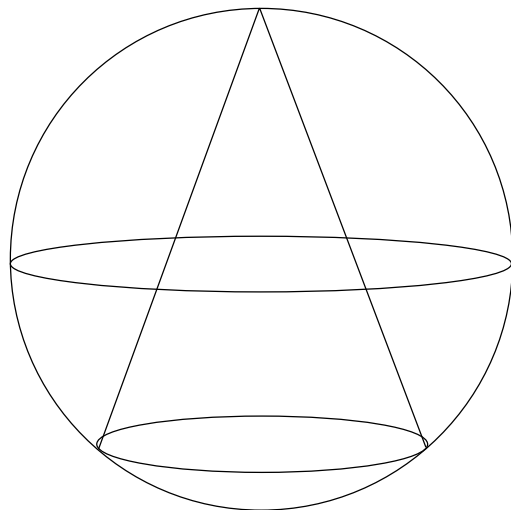
10) What is the Mean Value Theorem? What is Rolle's Theorem? Prove one of these two results.

11) What is the Fundamental Theorem of Calculus? Why is it true?

§3 Applications:

12) Six feet of wire is used to form a square and a circle. How much wire should be used for the square and how much should be used for the circle to enclose the maximum possible area? What if we want the minimum area?

13) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r .



14) A conical tank, with vertex down, is 10 feet across at the top, and 12 feet deep. If water is flowing into the tank at the rate of 10 cubic feet per minute, find the rate of change of the height of the water at the moment when the water is 8 feet deep.

15) A student driving home has come to a complete stop at a traffic light. If, after the light turns green, the student accelerates at a rate of $a(t) = \frac{75}{(t+1)^2}$ feet per second per second. After 10 seconds, how fast is our student traveling, and how far have they gone?