

Exam #2
Math 274
October 18, 2000

Name _____

Graphing calculators are permitted. All work must be shown. Do all work on the blank paper provided.

§1 Calculation

1. Evaluate $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$.
2. Evaluate $\int e^x \cos x \, dx$.
3. Evaluate $\int \tan^3 x \, dx$.
4. Evaluate $\int \frac{\sqrt{9-x^2}}{x^2} dx$.
5. Evaluate $\int \frac{dx}{x^3+x}$.
6. Use Simpson's rule with 6 intervals to approximate $\int_0^1 e^{x^2} dx$.

§2 Comprehension

7. What is the definite integral? Give a *precise* definition.
8. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{2} \frac{\pi}{n} \left[\sin 0 + 2 \sin\left(\frac{\pi}{n}\right) + 2 \sin\left(\frac{2\pi}{n}\right) + 2 \sin\left(\frac{3\pi}{n}\right) + \cdots + 2 \sin\left(\frac{(n-1)\pi}{n}\right) + \sin \pi \right]$ exactly.

§3 Application

9. Express the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as an integral. Evaluate the integral, and determine the area.
10. Newton's Law of Gravitation states that two bodies with masses m and M attract each other with a force $F = G \frac{mM}{r^2}$ where r is the distance between the bodies and G is the gravitational constant.
Assume a object of mass m is on the surface of a planet of mass M and radius r . How much work is needed for that satellite to escape the gravitational effects of the planet, or equivalently, how much work is needed to move the satellite an infinite distance from the planet?

The *escape velocity* v is the velocity needed to propel an object out of the gravitational field of a planet. Determine the escape velocity for our planet. Use the fact that the initial kinetic energy $\frac{1}{2}mv^2$ of the object supplies all of the work needed.