

**Midterm Exam**  
Math 274  
November 16, 2000

Name \_\_\_\_\_

*Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam. The use of a graphing calculator is permitted.*

**§1 Calculation**

1. Find the length of the curve  $y = \ln \cos x$  from  $x = 0$  to  $x = \pi/4$ .
2. The portion of the arc of the curve  $y = \ln x$  lying in the fourth quadrant is revolved about the  $y$ -axis. Find the area of the surface generated.
3. Find the (exact) center of mass of a semicircular plate of radius  $r$ .
4. Use Euler's method with a step size of 0.1 to estimate  $y(0.5)$ , where  $y(t)$  is the solution to the initial value problem  $y'(t) = t + y$ ,  $y(0) = 1$ .
5. Find the solution of the initial value problem 
$$\begin{cases} y' = x^2 y, \\ y(0) = 2. \end{cases}$$

**§2 Comprehension**

6. Let  $f(x)$  be a function with continuous derivative, but not necessarily positive. What is the formula for area of the surface formed by rotating the portion of the curve between  $x = a$  and  $x = b$  about the  $x$ -axis?
7. What is the direction field for a differential equation? Sketch the direction field for  $y' = y(2 - y)$ , and draw in some representative solutions.

**§3 Application**

8. Find the hydrostatic force on one end of a cylindrical drum with radius 3 ft if the drum is submerged in water 10 ft deep. Note: water weighs 62.5 lbs/ft<sup>3</sup>.
9. A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 30 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after  $t$  minutes?
10. How long will it take an investment to double in value if the interest rate is 6% compounded continuously? How does the time required change if you double the interest rate?