Exam #2
Math 274
October 15, 2003

Name_____

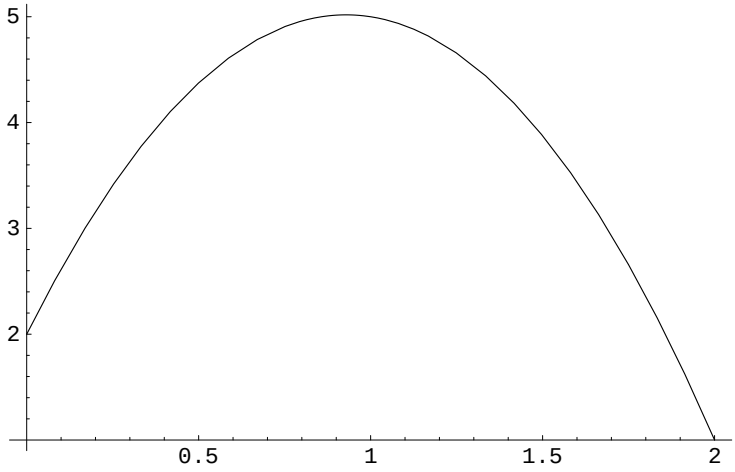
All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Evaluate $\int x \sin x \, dx$
2. Evaluate $\int_0^{\pi} \sin^2 x \, dx$
3. Evaluate $\int \cot^4 x \, dx$
4. Evaluate $\int_0^1 \dfrac{dx}{x^2 + 5x + 6}$
5. Evaluate $\int \dfrac{dx}{x^2\sqrt{4 - x^2}}$
6. Evaluate $\int_0^2 \dfrac{dx}{(x-1)^{2/3}}$
7. Evaluate $\int \dfrac{dx}{2 - \sqrt{3x}}$.

§2 Comprehension

8. Below is the graph of a function $f(x)$. For the integral $\int_0^2 f(x) \, dx$, find the approximations T_2, M_2, and S_2. Give an estimate of the integral, including an estimate of the error.



9. How high should you take n to ensure that S_n approximates $\int_0^1 e^{-x^2}\,dx$ to within 10^{-6}. (You do not need to simplify your answer.)

§3 Application

10. Consider a planet with mass M and radius R. The force of gravity F that the planet exerts on an object of mass m is $F = G\frac{mM}{x^2}$ where G is the universal gravitational constant, and x is the distance between the object and the center of the planet. How much work is needed to move the object infinitely far away from the planet?

    The escape velocity of a planet is the velocity v_0 needed to propel an onject to infinity. Use the fact that the intial kinetic energy $K = \frac{1}{2}mv_0^2$ provides the needed work to determine the escape velocity of a planet.