

Exam #3
Math 274
November 10, 2003

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

§1 Calculation

1. Find the length of the portion of the curve $y^2 = x^3$ between $(0, 0)$ and $(4, 8)$.
2. Find the area of the surface formed when the portion of curve $y = 1 - x^2$ for $0 \leq x \leq 1$ is rotated about the y -axis.
3. Sketch the direction field for the differential equation $y' = \frac{1}{2}(x^2 + y^2) - 1$. Draw the solution through $(0, 0)$. Draw the solution through $(0, 1)$. Draw the solution through $(0, -1)$.
4. Consider the problem

$$\begin{cases} y'(t) = y^2 - t - 1 \\ y(0) = 1 \end{cases}$$

Use Euler's method with a step size of 0.25 to find an approximation to $y(1)$.

5. Solve the differential equation

$$\begin{cases} x + 2y\sqrt{x^2 + 1} \frac{dy}{dx} = 1 \\ y(0) = 2 \end{cases}$$

6. Evaluate the limits, or explain why they do not exist.

a. $\lim_{n \rightarrow \infty} \frac{\ln n}{n + 1}$

b. $\lim_{n \rightarrow \infty} \frac{(-1)^n}{n + 1}$

c. $\lim_{n \rightarrow \infty} \frac{n^n}{n!}$

§2 Comprehension

7. Give an informal definition of the statement $\lim_{n \rightarrow \infty} a_n = L$. Give a precise definition. Use the precise definition to prove that $\lim_{n \rightarrow \infty} 2 - \frac{1}{n} = 2$.

§3 Application

8. The half life of ^{25}Na is 15 hours. How long will it take for 80% of a sample of this nuclide to decay?
9. Find the center of mass of the plane lamina bounded by the curves $y = \ln x$, $y = 0$, and $x = e$.
10. Find the force exerted by the water on a circular door with radius 3 ft if the door is 10 ft below the surface of the water. Water weighs 62.5 lbs/ft^3 .

