

Final Examination

Math 274

December 15, 2003

Name _____

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work.

Show all of your work. Work must be shown to receive full credit!

§1 Calculation

1. Evaluate $\int_0^1 x^2 \sqrt{1 + 2x^3} dx$

2. Evaluate $\int_0^1 \frac{dx}{x^2 + 5x + 6}$

3. Evaluate $\int_0^2 \frac{dx}{(x-1)^{2/3}}$

4. Consider the integral $\int_0^1 e^{-x^2} dx$.

a. Use power series to evaluate the integral exactly.

b. Using either power series or a numerical technique, approximate the integral accurately to 0.001. Explain your method, and be sure to explain why you are guaranteed to be sufficiently accurate.

Hint: If $f(x) = e^{-x^2}$, then $f'(x) = -2xe^{-x^2}$, $f''(x) = (4x^2 - 2)e^{-x^2}$,
 $f'''(x) = (12x - 8x^3)e^{-x^2}$ and $f''''(x) = (16x^4 - 48x^2 + 12)e^{-x^2}$

5. Solve the differential equation

$$\begin{cases} x + 2y\sqrt{x^2 + 1} \frac{dy}{dx} = 1 \\ y(0) = 2 \end{cases}$$

6. Consider the problem

$$\begin{cases} y'(t) = y^2 - t - 1 \\ y(0) = 1 \end{cases}$$

Use Euler's method with a step size of 0.25 to find an approximation to $y(1)$.

For problems 7–8, prove that the series either converges absolutely, converges conditionally, or diverges.

$$7. \sum_{n=1}^{\infty} \frac{1}{2^n + 1}$$

$$8. \sum_{n=1}^{\infty} \frac{(-1)^n(6n^2 - 9n + 4)}{n^3}$$

9. Show that the function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{n!}$ satisfies $f'(x) + 3x^2 f(x) = 0$. Find the radius of convergence of this series. Find the domain of the function f .

§2 Comprehension

10. What is the fundamental theorem of calculus? Use it to evaluate $\frac{d}{dx} \int_0^x \frac{ds}{1+s^3}$.

11. What is the precise meaning of $\sum_{k=1}^{\infty} c_k$?

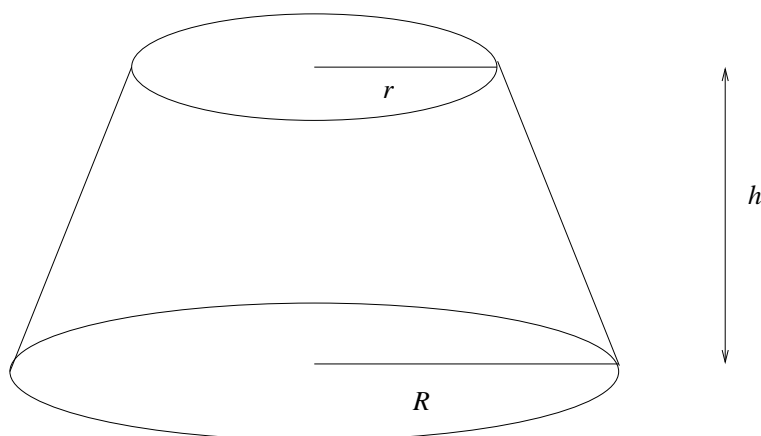
12. What is a geometric series? When is it convergent? When convergent, to what does it converge? Prove your results!

13. State Taylor's Theorem. What is the Maclaurin series for $\sin x$? Use Taylor's Theorem to prove that your answer is correct.

§3 Application

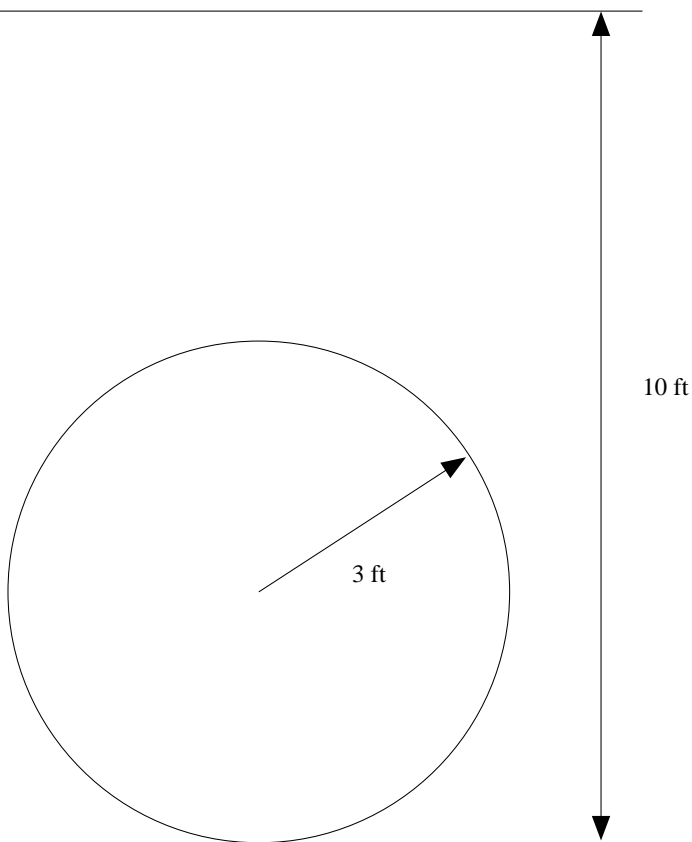
14. Find the area of one component of the region between $y = \cos^2 x$ and $y = 1 - 2 \sin^2 x$.

15. Find the volume of the frustum of a right circular cone with height h , lower base radius R , and upper base radius r .



16. Find the center of mass of the plane lamina bounded by the curves $y = \ln x$, $y = 0$, and $x = e$.

17. Find the force exerted by the water on a circular door with radius 3 ft if the door is 10 ft below the surface of the water. Water weighs 62.5 lbs/ft³.



18. The electric potential V at a distance R along the axis perpendicular to the center of a charged disc with radius a and charge density σ is given by

$$V = 2\pi\sigma(\sqrt{R^2 + a^2} - R).$$

If R is much larger than a , show that

$$V \approx \frac{\pi a^2 \sigma}{R}.$$

Estimate the error.