

Midterm Exam #1
Math 275
September 26, 2001

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. A curve C is defined by the parametric equations $x = t^2$, $y = t^3 - 3t$.
 - a. Show that C has two tangents at the point $(3,0)$ and find their equations.
 - b. Find the points on C where the tangent is horizontal or vertical.
 - c. Sketch the curve.

2. Find the (exact) area of the region that lies inside the curve $r = 3 \sin \theta$ and outside the curve $r = 1 + \sin \theta$.

3. Let $\mathbf{a} = \langle 1, -1, 3 \rangle$ and $\mathbf{b} = \langle 2, 1, 5 \rangle$. Find
 - a. $|\mathbf{a}|$,
 - b. $\mathbf{a} \cdot \mathbf{b}$,
 - c. $\mathbf{a} \times \mathbf{b}$,
 - d. the exact angle between \mathbf{a} and \mathbf{b} ,
 - e. a unit vector in the same direction as \mathbf{a} .

4. Find the equation of the plane through $P(1,3,2)$, $Q(3,-1,6)$, and $R(5,2,0)$.

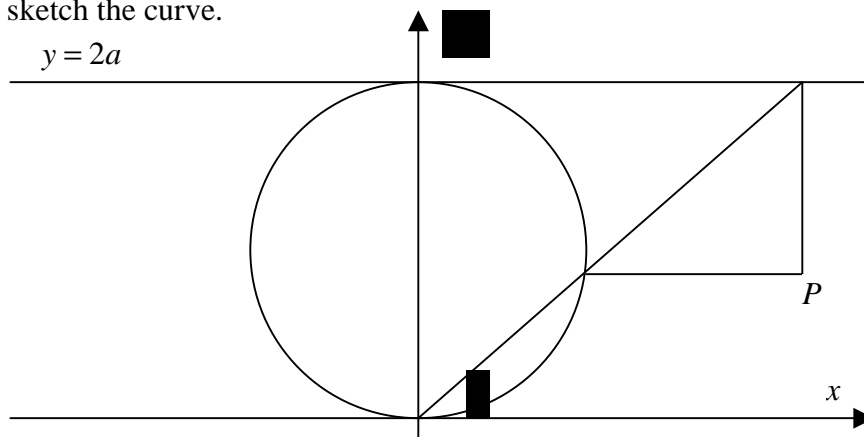
§2 Comprehension

5. What is the projection of the vector \mathbf{b} on the vector \mathbf{a} ? What is the component of the vector \mathbf{b} along the vector \mathbf{a} ? Explain your answer algebraically and geometrically.

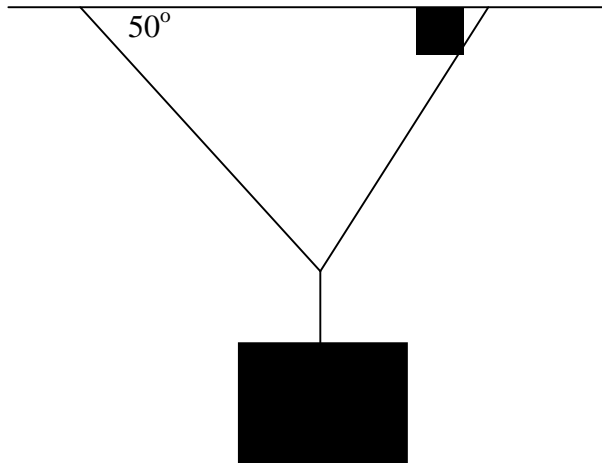
6. Is $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$? Explain. Give a formula for $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ that only uses the dot product.

§3 Application

7. A curve consists of all points P determined as specified in the picture. Find the parametric equations of the curve, and sketch the curve.



8. A 200-lb weight hangs from two wires as shown in the figure. Find the (approximate) magnitude of the tensions (forces) in both wires.



9. Let P be a point not on the line L that passes through the points Q and R . Express the distance between the point P and the line L using only the vectors $\mathbf{a} = \overrightarrow{QR}$ and $\mathbf{b} = \overrightarrow{QP}$.
10. Consider the planes $x + y - z = 2$ and $3x - 4y + 5z = 6$. Prove that the planes intersect. Find the parametric equations of the line of intersection, and find the (exact) angle between the planes.