Midterm Exam #3
Math 275
November 7, 2001

Name__________________________

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ or show that the limit does not exist.

2. Find the first partial derivatives of the function $f(x,y,z) = xy^2z^3 - 4yz$. Also, find f_{xyz}.

3. Suppose $x^3 + y^3 + z^3 - 6xyz = 1$ defines $z(x,y)$ implicitly as a function of x and y. Find $\frac{\partial z}{\partial x}$ and z_y.

4. Find an equation for the tangent plane to the surface $x^3y^2 - 5x - z = 0$ at the point (1,2,-1).

§2 Comprehension

5. What is the gradient vector? What is the relationship between the gradient vector and the level curves of a function? Draw the contour graph of some function (your choice) and draw in the gradient vector to your function at four representative points.

6. What is the directional derivative of $f(x,y)$ in the direction $\langle a,b\rangle$? What condition does the vector $\langle a,b\rangle$ need to satisfy? What is the relationship between the directional derivative and the gradient vector?

§3 Application

7. If $z = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, write $\frac{\partial^2 f}{\partial\theta^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)$ in terms of f_{xx}, f_{yy}, f_{xy}, f_x, and f_y.

8. Suppose that you are climbing a hill whose shape is given by the equation $z = 1000 - \frac{x^2}{100} - \frac{y^2}{50}$ and you are standing at the point with coordinates $(60,100,764)$.
    a. In which direction should you proceed initially in order to reach the top of the hill fastest?
    b. If you climb in that direction, at what angle above the horizontal would you be climbing initially? (Give an approximate answer, in degrees.)

9. Find the absolute extrema of the function $f(x,y)=2x^3+y^4$ on the set $D=\{(x,y)\mid x^2+y^2\le 1\}$.

10. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z)=8x-4z$ subject to the constraint $x^2+10y^2+z^2=5$.