Final Exam
Math 275
December 18, 2001

Name_____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

Solve only 16 of the 18 problems provided. *Only 16 problems will be graded. If more than 16 problems are solved, I will choose which 16 to grade, probably to your detriment. Indicate which two problems you skipped:*

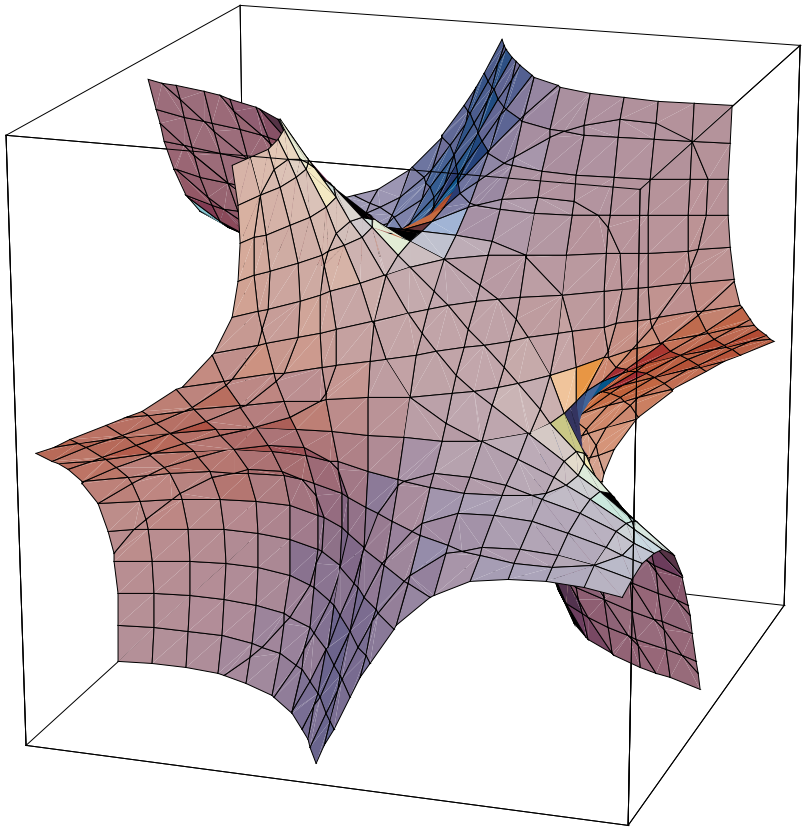
Skipped Problems _____ _____.

§1 Calculation

1. Find the equation of the plane through $P(1,2,1)$, $Q(0,2,0)$, and $R(1,1,0)$. Find the distance from this plane to the point $S(-1,-2,3)$.

2. Let $f(x,y) = xe^y$. Find ∇f. What is the rate of change of f at the point $P(2,0)$ in the direction from P to $Q(\frac{1}{2},2)$? In what direction does f change most rapidly? What is this maximum rate of change?

3. The surface $x^3 + y^3 + z^3 + 6xyz = 22$ is graphed below. Find the equation of the tangent plane at $(1,1,2)$.



4. Evaluate $\int_0^1 \int_x^1 \sin y^2 \, dy \, dx$ exactly.

5. Find the exact volume of the solid bounded by the cylinder $x^2 + z^2 = 4$ and the planes $y = 0$ and $y = z + 2$.

6. Evaluate $\iiint_R x \, dV$ exactly, where R is the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$.

7. Evaluate $\iint_R e^{(x+y)/(x-y)} dA$ exactly, where R is the trapezoid bounded by $(1,0)$, $(2,0)$, $(0,-2)$, and $(0,-1)$.

8. Evaluate $\int_C y \, dx + x \, dy$ exactly where C is the unit circle, traversed counterclockwise.

§2 Comprehension

9. Let P be a point not on the line L that passes through the points Q and R. Show that the distance d from the point P to the line L is $d = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$, where $\mathbf{a} = \overrightarrow{QR}$ and $\mathbf{b} = \overrightarrow{QP}$. Be sure to clearly explain your reasoning. Find the distance from the point $P(1,1,1)$ to the line through $Q(0,6,8)$ and $R(-1,4,7)$.

10. What is the precise definition of curvature? In the process, define all of the terms that you use. Show that the curvature of a circle of radius a is $1/a$.

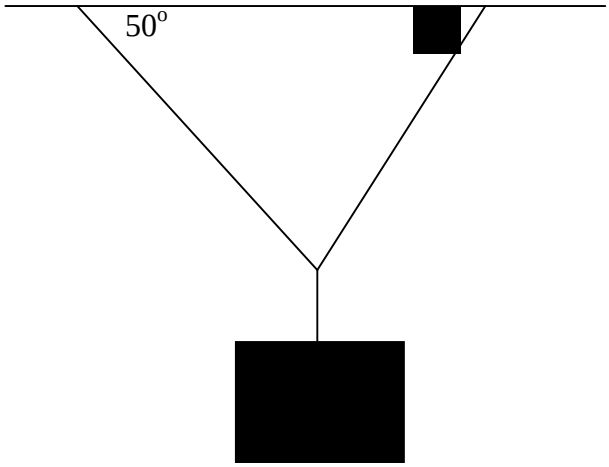
11. Let $z = f(x + at) + g(x - at)$, for some functions f and g. Find $\dfrac{\partial^2 z}{\partial t^2} - a^2 \dfrac{\partial^2 z}{\partial x^2}$, and simplify your answer.

12. What is the gradient vector? What is the relationship between the gradient vector and the level curves of a function? Draw the contour graph of some function (your choice) and draw in the gradient vector to your function at four representative points.

13. Let $\mathbf{F} = \dfrac{\langle -y, x \rangle}{x^2 + y^2}$, and let C be a piecewise smooth simple closed curve that does not cross the origin. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. How does your answer depend on C?

§3 Application

14. A 200-lb weight hangs from two wires as shown in the figure. Find the (approximate) magnitude of the tensions (forces) in both wires.



15. A projectile is fired with an initial speed of 500 m/s and angle of elevation 30^o. Find the (approximate) (horizontal) range of the projectile.

16. Find the absolute extrema of the function $f(x,y) = 2x^3 + y^4$ on the set $D = \{(x,y) \mid x^2 + y^2 \le 1\}$.

17. Find the volume and center of mass of the solid E of unit density that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

18. Let $f(\mathbf{x}) = \dfrac{mMG}{|\mathbf{x}|}$, and let $\mathbf{F} = \dfrac{-mMG}{|\mathbf{x}|^3}\mathbf{x}$. Show that $\mathbf{F} = \nabla f$, and find the work done by the gravitational field $\mathbf{F}$ in moving a particle with mass m from the point $(3,4,12)$ to the point $(2,2,0)$ along a piecewise smooth curve.