Exam #2
Math 275
March 14, 2000

Name__________________________

§1 Calculation

1. Convert each of the following from the indicated coordinate system to Cartesian coordinates, and graph the result. Identify the resulting surface.
   a. $r^2 = z$ (cylindrical)
   b. $\rho = 4\sec\phi$ (spherical)

2. Let $\mathbf{r}(t) = t^2\mathbf{i} + 3t\,\mathbf{j} + \dfrac{1-\cos t}{t}\mathbf{k}$. Find $\lim_{t\to 0}\mathbf{r}(t)$.

3. Let $\mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \ln t\,\mathbf{k}$. Find the domain of $\mathbf{r}(t)$, and determine the interval(s) on which $\mathbf{r}(t)$ is continuous.

4. Let $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, t\rangle$. Find $\mathbf{T}(t)$ and the parametric equations of the line tangent to $\mathbf{r}(t)$ when $t = \pi/4$.

5. Find the curvature of $\mathbf{r}(t) = \langle 2t, t^2, -\tfrac{1}{3}t^3\rangle$.

§2 Comprehension

6. What is the definition of the derivative $\dfrac{d}{dt}\mathbf{r}(t)$? Include an example. What is the geometric relationship between $\dfrac{d}{dt}\mathbf{r}(t)$ and $\mathbf{r}(t)$?

7. Suppose that $\|\mathbf{r}(t)\| = 1$. Prove that $\mathbf{r}(t)$ is perpendicular to $\mathbf{T}(t)$, and that $\mathbf{T}(t)$ is perpendicular to $\mathbf{N}(t)$.

§3 Application

8. At time $t = 0$, a particle is a height h above the ground, and is moving with speed v_0 at an angle θ with the horizontal. If the acceleration of the particle is $\mathbf{a}(t) = \langle 0, -g\rangle$, find the path $\mathbf{r}(t)$ that the particle will follow.

9. A baseball is hit three feet above the ground at 100 feet per second, at an angle of 45° with respect to the ground. Find the maximum height reached by the ball. Will the ball clear a 10-foot high fence located 300 feet away? [The acceleration due to gravity is 32 feet per second per second.]

10. The Cornu spiral is the curve $\mathbf{r}(t) = \left\langle \int_0^t \cos\frac{\pi s^2}{2}\,ds, \int_0^t \sin\frac{\pi s^2}{2}\,ds, \right\rangle$ and on the interval $[-2\pi, 2\pi]$ has the graph given below. Find the length of this curve from $t = 0$ to $t = a$. Find the curvature of $\mathbf{r}(t)$ when $t = a$. What is the relationship between the length and curvature? How is this relationship reflected in the curve?