

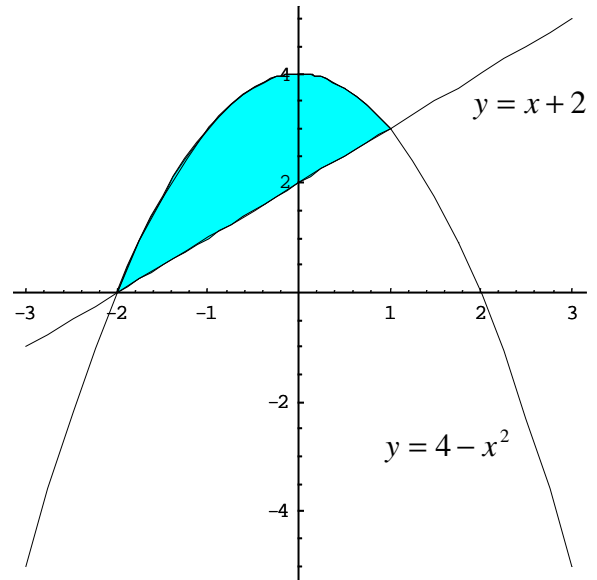
Exam #4
Math 275
May 11, 2000

Name _____

Do all of your work on the blank paper that has been provided.

§1 Calculation

1. Evaluate $\int_1^2 \int_1^x \left(\frac{2x^2}{y^2} + 2y \right) dy dx$ exactly.
2. Write a double integral that gives the area of the shaded region in the figure. Use it to find the (exact) area of the shaded region.
3. Find the (exact) volume of the region bounded by $f(x, y) = e^{-x^2}$, and the planes $y = 0$, $x = 1$, and $y = x$.
4. Find the (exact) volume of the region bounded above by $z = \sqrt{16 - x^2 - y^2}$, and below by the xy -plane, within the cylinder $x^2 + y^2 = 9$.



5. Change the order of integration, and evaluate $\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \int_1^4 \sin(y^2) dz dy dx$ exactly.

§2 Comprehension

6. What is an iterated integral? What is a double integral? What is the relationship between them?
7. What is the Jacobian change of variables? Find the Jacobian $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$ for cylindrical coordinates, and the Jacobian $\frac{\partial(x, y, z)}{\partial(\rho, \theta, \phi)}$ for spherical coordinates.

§3 Application

8. Find the (exact) center of mass of the plane lamina bounded by $y = \sqrt{a^2 - x^2}$, $y = 0$, and $x = 0$, where the density at each point is proportional to the distance from the point to the origin.
9. Find the (exact) surface area of the portion of the surface given by $z = 2 + x^{3/2}$ that lies over the rectangle with vertices at $(0,0)$, $(0,4)$, $(3,4)$, and $(3,0)$.
10. Find the (exact) center of mass of the solid of uniform density bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$, and above by the sphere $x^2 + y^2 + z^2 = a^2$.