Math 675
Assignment #2
Due September 21, 2009

Name______________________________

12. For each of the following, determine and classify all of the singular points.
    (a) $y'' = xy$
    (b) $x^2y'' + xy' + (x^2 - \nu^2)y = 0$
    (c) $x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0$
    (d) $xy'' + (b-x)y' - ay = 0$.
13. Solve $y'' - x^2y = 0$ in the form of a series at the origin.
14. Prove Euler's Formula $e^{i\theta} = \cos\theta + i\sin\theta$ for real θ. Continue and show the following for $z = x + iy$
    (a) $\sin z = \sin x \cosh y + i \cos x \sinh y$
    (b) $\cos z = \cos x \cosh y - i \sin x \sinh y$
    (c) $\sinh z = \sinh x \cos y + i \cosh x \sin y$
    (d) $\cosh z = \cosh x \cos y + i \sinh x \sin y$
15. For each of the following, determine if the equation admits of a series solution in the form $y = \sum_{n=0}^{\infty} a_n x^n$. If it does, give a lower bound for the radius of convergence of the series.
    (a) $(x+1)y'' + (x+2)y' + (x+3)y = 0$
    (b) $(\sin x)y'' + (\cos x)y' - (\sin x)y = 0$
    (c) $(\sinh x)y'' + (\cosh x)y' - (\sinh x)y = 0$
16. For $z \in \mathbf{C}$ with $\Re z > 0$, the gamma function is defined by

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt.$$

Show that, for real x with $x > 0$, $\Gamma(x+1) = x\Gamma(x)$. Conclude that, for all $\alpha > 0$

$$\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}.$$

17. Find one solution as a Frobenius series abouth the origin for Bessel's equation

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0.$$