Math 675
Assignment #3
Due September 28, 2009

Name______________________________

18. Find a second solution as a Frobenius series about the origin for Bessel's equation

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0$$

when ν is not an integer.

19. Find a second solution to Bessel's equation of order ν for $\nu = 0$.

20. Prove the following: If $f = o(g)$, $x \to x_0$, then $f = O(g)$, $x \to x_0$.

21. True or false. In each case, justify your answer.

    (a) $x \ll 1/x$, $x \to 0^+$.
    (b) $x \ll 1/x$, $x \to \infty$.
    (c) $\sin x \ll -2$, $x \to 0$.
    (d) $\sqrt{x} + 1/\sqrt{x} \sim \sqrt{x}$, $x \to \infty$.
    (e) $\cos x \sim 0$, $x \to \pi/2^-$

22. Prove the following: If $f(x) \sim g(x)$, $x \to x_0$, then $g(x) \sim f(x)$, $x \to x_0$.

23. Prove each of the following:

    (a) $\log x = o(x^{-p})$, $x \to \infty$, for all $p > 0$.
    (b) $\sin x = O(x)$, $x \to 0$.
    (c) $e^{-1/x} = O(x^p)$, $x \to 0$, for all $p > 0$.

24. Prove each of the following:

    (a) If $f = O(g)$, $x \to x_0$, then $f = O(-g)$, $x \to x_0$.
    (b) If $f = o(g)$, $x \to x_0$, then $f = o(-g)$, $x \to x_0$.

25. Prove each of the folowing:

    (a) If $f_n = O(g_n)$, $x \to x_0$ for $n = 1, 2, \ldots, N$, then for any $\lambda_1, \lambda_2, \ldots, \lambda_N$ we have

    $$\sum_{n=1}^{N} \lambda_n f_n = O\left(\sum_{n=1}^{N} |\lambda_n||g_n)\right)$$

    as $x \to x_0$.

    (b) If $f = o(g)$, $x \to x_0$, then

    $$\int_0^x f(t)\,dt = o\left(\int_0^x |g(t)|\,dt\right)$$

    as $x \to x_0$.

26. Show that $x \sin(1/x) = O(x)$ as $x \to 0$. Is it the case that

$$\frac{d}{dx}\left(x \sin \frac{1}{x}\right) = O\left(\frac{d}{dx} x\right) = O(1)$$

as $x \to 0$?

27. Prove the following: If $f(x)+g(x) = h(x)$, and $f(x) \ll g(x)$, $x \to x_0$, then $g(x) \sim h(x)$, $x \to x_0$.

28. Prove the following: If $f(x) \sim a(x-x_0)^{-b}$, $x \to x_0+$ for $b > 1$, then

$$\int^x f(t)\,dt \sim \frac{a}{1-b}(x-x_0)^{1-b}$$

for $x \to x_0+$.

29. Prove the following: If $f(x) \sim a(x-x_0)^{-1}$, $x \to x_0+$, then

$$\int^x f(t)\,dt \sim a\ln(x-x_0)$$

for $x \to x_0+$.

30. Prove the following: If $f(x) \sim a(x-x_0)^{-b}$, $x \to x_0+$ for $b < 1$, then

$$\int^x f(t)\,dt \sim c$$

for $x \to x_0+$, for some constant c.

31. Prove the following: If $f(x) \sim a(x-x_0)^{-b}$, $x \to x_0+$ for $b < 1$, then

$$\int_{x_0}^x f(t)\,dt \sim \frac{a}{1-b}(x-x_0)^{1-b}$$

for $x \to x_0+$.