

**Math 675**  
Assignment #5  
Due October 12, 2009

Name \_\_\_\_\_

38. Find an asymptotic power series expansion for

(a)  $f_1(x) = \frac{1}{1+x}$ .

(b)  $f_2(x) = \frac{1}{1+x} + e^{-1/x}$ .

as  $x \rightarrow 0$ . What can you conclude?

39. Find the first three terms in the asymptotic power series expansion as  $x \rightarrow 0$  of  $f(x) = \sin(e^x)$ .

40. The function

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

is important in probability, statistics, and in many other areas.

(a) Expand the integrand in series about  $t = 0$  and integrate to obtain a series expansion for  $\operatorname{erf}(z)$ .

(b) How many terms in the series in (a) are required to evaluate  $\operatorname{erf}(5)$  accurately to  $10^{-5}$ ?

(c) Verify

$$\operatorname{erf}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$$

(d) Repeatedly integrate  $e^{-t^2}$  by parts to obtain a series for  $\operatorname{erf}(z)$  in powers of  $z^{-1}$  multiplied by  $e^{-z^2}$ .

(e) Verify that the series in (d) is an asymptotic series.

(f) How many terms are required to estimate  $\operatorname{erf}(5)$  to  $10^{-5}$ .

41. Find the leading behavior as  $x \rightarrow \infty$  of both solutions to  $y'' = \sqrt{x}y$ .

42. Find the leading behavior as  $x \rightarrow \infty$  the solutions to  $y'' = (\cosh x)y'$ .

43. Find the leading behavior as  $x \rightarrow \infty$  of both solutions to  $x^2y'' + xy' - (x^2 + \nu^2)y = 0$ .

44. Find the first few terms in the local behavior as  $x \rightarrow 0^+$  of a particular solution to  $y' + xy = x^{-3}$ .

45. Find the leading behavior as  $x \rightarrow \infty$  of a particular solution to  $y'' + xy = x^5$ .

46. Find the first few terms in the behavior as  $x \rightarrow \infty$  of  $y'' + y/x^3 = x$ .